SOLUTION

Problem 1

a)

SWOF – 2-phase water/oil data

- 1. column: water saturation
- 2. column: water rel. perm.
- 3. column: oil rel. perm.
- 4. column: water/oil capillary pressure

SGOF – 2-phase gas/oil data

- 1. column: gas saturation
- 2. column: gas rel. perm.
- 3. column: oil rel. perm.
- 4. column: gas/oil capillary pressure

b)

- 3-phase water rel. perm: water rel. perm from SWOF table
- 3-phase gas rel. perm: gas rel. perm from SGOF table
- 3-phase oil rel. perm: a combination of the relative permeabilities specified in tables SWOF and SGOF. One Example is Baker's formula

$$k_{ro}(S_w, S_g) = \frac{(S_w - S_{wr})k_{row}(S_w) + S_g k_{rog}(S_g)}{S_g + S_w - S_{wr}}$$

where k_{row} and k_{rog} are oil rel. perm. measured in 2-phase water/oil and oil/gas systems, S_{wr} critical water saturation.

c)

- 4. column in SWOF table must consist of decreasing numbers
- In 1. column of SWOF numbers are not increasing
- In 2. column of SGOF numbers are not increasing.
- In 4. column of SWOF numbers are not decreasing.

Problem 2

a)

Taylor expansions

$$u(x-\Delta x) = u(x) - u'(x)\Delta x + \frac{1}{2!}u''(x)\Delta x^2 - \frac{1}{3!}u'''(c1)\Delta x^3 \qquad (1)$$

$$u(x-2\Delta x) = u(x) - 2u'(x)\Delta x + 2u''(x)\Delta x^2 - \frac{8}{3!}u'''(c2)\Delta x^3 \qquad (2)$$

Combine equations (2) – 4*(1)

$$u(x-2\Delta x) - 4u(x-\Delta x) = -3u(x) + 2u'(x)\Delta x + R\Delta x^3$$

i.e.

$$u_x = \frac{u_{i-2}-4u_{i-1}+3u_i}{2\Delta x} + O(\Delta x^2)$$

b)

The error function $\varepsilon_i^n = \xi^n e^{Ji\phi}$ substituted into the difference equation gives

$$\gamma\xi^n e^{J(i-1)\varphi} - 2\gamma\xi^n e^{Ji\varphi} + \gamma\xi^n e^{J(i+1)\varphi} = \xi^{n+1}e^{Ji\varphi} - \xi^n e^{Ji\varphi},\ \gamma = \frac{\Delta t}{\Delta x^2}$$

and dividing by $e^{Ji\phi}$ gives

$$\gamma(e^{-J\phi} - 2 + e^{J\phi}) = \xi - 1.$$

The left hand side can be written $2\gamma(\cos\phi - 1) = -4\gamma\sin^2(\phi/2)$, and hence,

$$\xi = 1 - 4\gamma\, sin^2(\varphi/2).$$

The von Neumann stability criterion

$$-1 \le 1 - 4\gamma\, sin^2(\varphi/2) \le 1$$

i.e. $\gamma \le 1/2$.

c)

The error function

$$\varepsilon_i^n = \xi^n e^{iJ\theta}$$

substituted in the difference equation gives

$$\xi^{n+1}e^{iJ\theta} - \xi^n e^{iJ\theta} = \gamma\xi^n e^{(i-1)J\theta} - \gamma\xi^n e^{iJ\theta}, \quad \gamma = \frac{\Delta t}{\Delta x}$$

Division by $\xi^n e^{iJ\theta}$:

$$\xi - 1 = \gamma(e^{-J\theta} - 1)$$
$$\xi = 1 - \gamma(1 - \cos\theta) - J\gamma \sin\theta$$
$$|\xi|^2 = \left[1 - \gamma(1 - \cos\theta)\right]^2 + \gamma^2 \sin^2\theta$$
$$= 1 + 2\gamma(1 - \cos\theta)(\gamma - 1).$$

Stability if $|\xi|^2 \le 1$, i.e.

$$2\gamma(1 - \cos\theta)(\gamma - 1) \le 0.$$

Requirement for stability: $\gamma \le 1$

Problem 3

a)

Time step length 1/16 gives $\gamma = \Delta t/\Delta x^2 = 1$ and formula for computing solution

$$u_i^{n+1} = u_{i-1}^n - u_i^n + u_{i+1}^n$$

Result.

Block	0	1	2	3	4
Step 0	0	1.5	3	1.5	0
Step 1	0	1.5	0	1.5	0
Step 2	0	-1.5	3	-1.5	0
Step 3	0	4.5	-6	4.5	0

b)

Time step length 1/40 gives $\gamma = \Delta t/\Delta x^2 = 0.4$ and formula for computing solution

$$u_i^{n+1} = 0.4u_{i-1}^n + 0.2u_i^n + 0.4u_{i+1}^n$$

Result.

Block	0	1	2	3	4
Step 0	0	1.5	3	1.5	0
Step 1	0	1.5	1.8	1.5	0
Step 2	0	1.02	1.56	1.02	0
Step3	0	0.828	1.128	0.828	0

c)

Stability analysis in 2b) shows unstable conditions for time step length 1/16 (see 3a) and stability for time step length 1/40 (see 3b). This is in agreement with results of calculations.

Problem 4

a)
Black Oil fluid data are specified using tables for viscosities, volume factors, solution ratio and reference densities of water, oil and gas.
For compositional computations an equation of state (EOS) is used to compute fluid properties. Hence, EOS parameters like critical temperatures and pressures are needed.

b)
In a compositional model mass balance is expressed in terms of moles for each component.

Problem 5

a)
Initial state is specified in SOLUTION section.
If EQUIL is used position of fluid contacts (depth to WOC and GOC) and a reference pressure must be specified. The reference pressure will normally be pressure in GOC contact.

b)
S_{wr} critical water saturation.
Below WOC $S_w = 1$, above WOC $S_w = S_{wr}$
Below GOC $S_g = 0$, above GOC $S_g = 1 - S_{wr}$
Finally $S_o = 1 - S_w - S_g$ in all blocks.

Problem 6

a)
Term for flow of oil between block i and i+1

$$F = C\left(\frac{k_{ro}}{B_o \mu_o}\right)_{i+\frac{1}{2}} (p_{i+1} - p_i)$$

where C is a constant.
Block i is upstream block

$$F = C\left(\frac{k_{ro}}{B_o \mu_o}\right)_{i} (p_{i+1} - p_i)$$

Block i is saturated (both oil and gas present) and hence k_{ro} is a function of two saturations.

$$F = C\frac{k_{ro}(S_{wi}, S_{gi})}{B_o(p_i)\mu_o(p_i)} (p_{i+1} - p_i)$$

Linearization k -> k+1.

$$\frac{\partial F}{\partial p_i}^k \Delta p_i^k + \frac{\partial F}{\partial p_{i+1}}^k \Delta p_{i+1}^k + \frac{\partial F}{\partial S_{wi}}^k \Delta S_{wi}^k + \frac{\partial F}{\partial S_{gi}}^k \Delta S_{gi}^k + F^k$$

b)
Accumulation term

$$F = C\left[\left(\frac{\varphi_i S_{oi}}{B_{oi}}\right)^{n+1} - \left(\frac{\varphi_i S_{oi}}{B_{oi}}\right)^{n}\right]$$

C is a constant and $A = \left(\frac{\varphi_i S_{oi}}{B_{oi}}\right)^{n}$ is constant as well. The term to be linearized is

$$F = C\left[\frac{\varphi_i S_{oi}}{B_{oi}} - A\right].$$

Since oil in block i is under saturated (no free gas) B_{oi} will be function of p_{si} and p_i.

$$F = C\left[\frac{\varphi(p_i) S_{oi}}{B_o(p_i, p_{si})} - A\right]$$

Linearization k -> k+1.

$$\frac{\partial F}{\partial p_i}^k \Delta p_i^k + \frac{\partial F}{\partial p_{si}}^k \Delta p_{si}^k + \frac{\partial F}{\partial S_{oi}}^k \Delta S_{oi}^k + F^k$$

Problem 7

a)
Outer iterations (non-linear, Newton) iterations to solve algebraic (discretized) equations.
Inner (linear) iterations to solve the large linear systems of equations during outer iterations.

b)
The linear equations will have block banded structure. The banded structure is caused by coupling to neighbor blocks.

- 1D: tridiagonal
- 2D: pentadiagonal
- 3D: heptadiagonal

The blocks are (nc+1)x(nc+1) matrices, nc the number of hydrocarbon component (nc+1 equations and unknowns for each grid block).

c)
For compositional simulations the number of components is large, frequently larger than 10. For implicit solution the block heptadiagonal linear systems with (nc+1)x(nc+1) matrices on the bands will require too large computer work.

The preferred method for compositional models is the AIM method where unknowns are treated implicitly only when needed for stability reasons.