UNIVERSITETET I STAVANGER
INSTITUTT FOR MATEMATIKK OG FYSIKK

MAF300 – Numerical Modelling
Exam (Theoretical part of the lecture), 2019

Candidate number

General instructions.

- You have 4 hours (09:00–13:00) to work on the test.
- Calculators allowed according to the faculty rules: *HP30S, all Casio FX82, Texas Instruments TI-30XB, Texas Instruments TI-30X Pro, Citizen SR-270X, Texas BA II Plus, HP17bII +, Citizen SR-260N and Casio FX85GT Plus*
- All printed or handwritten material is allowed, including the lecture notes and the books used during the course.
- There are six problems covering the material you have learned during the lectures.
- The maximum number of points you can obtain for a completely correctly solved problem is indicated in parentheses after the problem number.

Finally some general recommendations:

- Read *all* questions carefully before you start working on the test!
- Read each question carefully again before answering it!
- Answer those questions that are easy for you first!
- Work out the solutions of the problems separately on the provided exam sheets! Remember that not only the final result, but also the way you obtained it will be graded! This means that in case your result is wrong, you can get partial credit for showing some understanding of the problem. It is therefore important to express your thoughts clearly.

UNIVERSITETET I STAVANGER
INSTITUTT FOR MATEMATIKK OG NATURVITENSKAP

MAF300 – Numerisk Modellering
Exam (Theoretical part of the lecture) – Fall 2019.

(A1) **Nonlinear equations** (20 points)
The equation

$$x^2 - e^{-2x} = 0 \tag{1}$$

has a root in the interval [0.4,0.8]. Find this root with an error less than 0.01 using different methods:

- the bisection method (use the given interval)
- the Newton-Raphson method (choose an initial value close to 0.6)

(A2) **Numerical differentiation** (15 points) Calculate the first derivative of the function

$$f(x) = x^{\frac{1}{3}} \tag{2}$$

at the point x=2 numerically using the following difference formulas from the lecture.

- Use the three–point forward difference formula for step widths h=0.2, h=0.1, h=0.01.
- Use the two–point central difference formula for step widths h=0.2, h=0.1, h=0.01.
- Use the four–point central difference formula for step widths h=0.2, h=0.1, h=0.01.
- Compare the result with the analytical solution and calculate the errors for all cases. Make a table listing the results and the errors for all cases and briefly discuss the quality of this result.

(A3) **Differential equations** (10 points)
Write the following system of two second-order ODEs as a system of four first-order ODEs:

$$\frac{d^2x}{dt^2} = -\frac{-2\alpha^3}{m}\left(\frac{dx}{dt}\right)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \tag{3}$$

$$\frac{d^2y}{dt^2} = -g - \frac{-2\alpha^3}{m}\left(\frac{dy}{dt}\right)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \tag{4}$$

(A4) **Systems of linear equations** (15 points)
Solve the following system of equations with LU decomposition using Crout's method

$$\begin{aligned} 2x - 6y + 6z &= 2 \\ 3x - 7y + 13z &= -13 \\ -2x + 2y - 11z &= 21 \end{aligned}$$

(A5) **Numerical integration** (20 points)
A wine barrel has the following diameters d at heights z:

z(in)	-18	-12	-6	0	6	12	18
d(in)	28	30	31	32	31	30	28

Use the composite Simpsons 1/3 method (compare the lecture notes on numerical integration) to determine the surface area $S = 2\pi \int_0^L r\,dz$ and the volume $V = \pi \int_0^L r^2\,dz$. Check first that the requirements for using this method are met, then calculate the integrals.

(A6) **Curve fitting** (20 points)
The approximate population of China for selected years from 1900 until 2010 is:

year	1900	1950	1980	1990	2000	2010
population (millions)	400	550	980	1130	1270	1390

- Assume that the population growth can be modeled with an exponential function

$$p = \alpha e^{m \cdot x} \tag{5}$$

  where x is the year and p is the population in millions. Write the equation in a linear form, then use linear least-squares regression to determine the constants α and m for which the function best fits the data.
- Use the equation to estimate the population in the years 1960 and 2030.