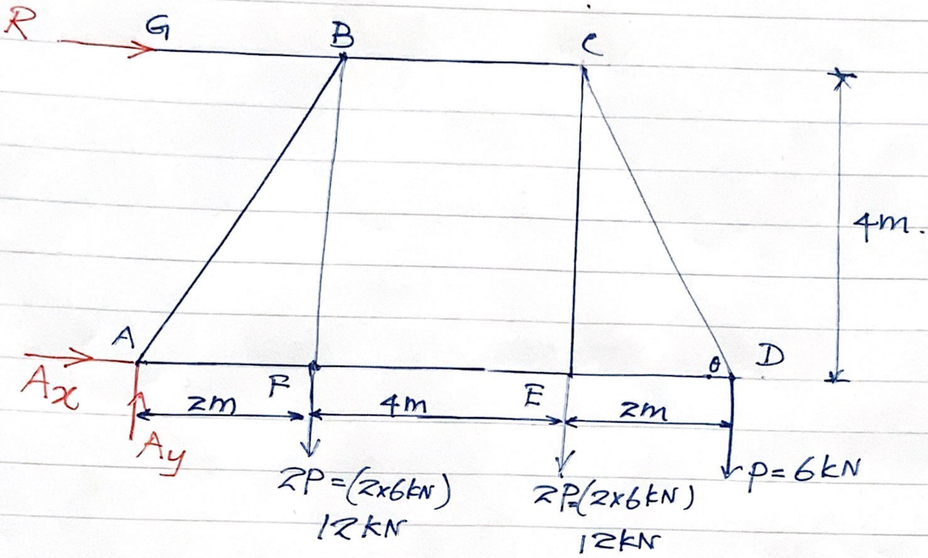
BYG 140 Konstruksjonsmekanikk 1

may 19, 2020

(Q1) (a)



Considering equilibrium,

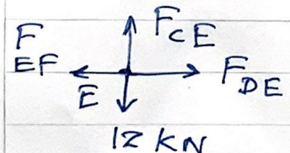
$\rightarrow \Sigma F_x = A_x + R = 0$

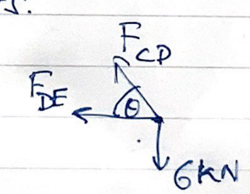
$\uparrow \Sigma F_y = A_y - 12 - 12 - 6 = 0 \qquad A_y = \underline{30 kN}$ ✓

$\curvearrowleft M_A = 4R + (12 \times 2) + (12 \times 6) + (6 \times 8) = 0 \qquad R = \underline{-36 kN}$ ✓

$A_x = -(-36) = \underline{36 kN}$ ✓

(b) Using method of joints.





$\sin\theta = \frac{4}{\sqrt{16+4}} = \frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{20}} = \frac{2}{2\sqrt{5}}$

$\underline{F_{CE} = 12 kN\ (T)}$ ✓

$\uparrow F_{CD}\sin\theta - 6 = 0$

$F_{CD} = 6/\frac{4}{\sqrt{20}} = \frac{6 \times 2\sqrt{5}}{4} = 6.71 kN.$

$\underline{F_{EF} = F_{DE} = 3 kN\ (C)}$ ✓

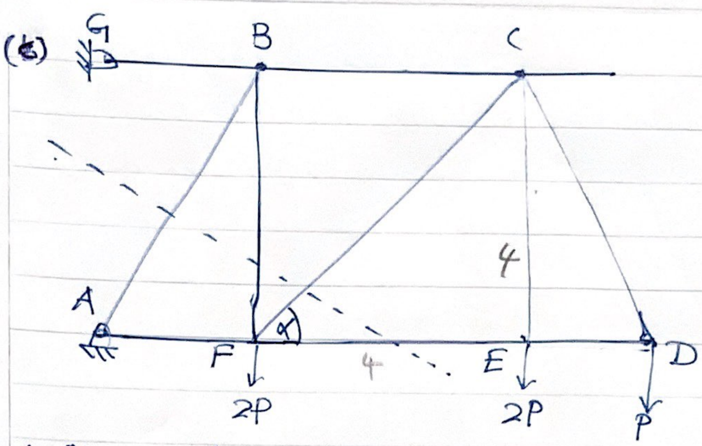
$\leftarrow F_{DE} + F_{CD}\cos\theta = 0$

$F_{DE} = -\frac{6 \times 2\sqrt{5}}{4} \times \frac{2}{2\sqrt{5}}$

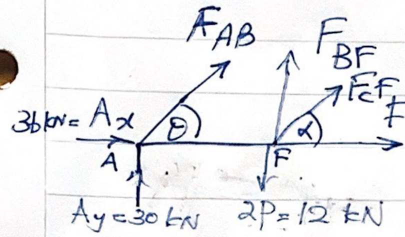
$F_{DE} = -3 kN$

$\underline{F_{DE} = 3 kN\ (C)}$ ✓

(c)



Using method of section.



F_{EF} = (−3kN − calculated in (a))

$\sin\alpha = \frac{4}{4\sqrt{2}}$

$\alpha = 45^\circ$

$\cos\alpha = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\rightarrow \Sigma F_x = F_{AB}\cos\theta + A_x + F_{EF} + F_{CF}\cos\alpha = 0$

$\circlearrowleft F = -F_{AB}\sin\theta \times 2 - (30\times 2) = 0$ → $15\sqrt{5}$

$F_{AB} = \frac{-30}{4/\sqrt{20}} = -33.54$ kN $= \underline{33.54\text{ kN (c)}}$

$F_{CF}\left(\frac{1}{\sqrt{2}}\right) = -(-33.54)\cdot\frac{2}{\sqrt{20}} - 36 - (-3) = -18$ kN $= \underline{18\sqrt{2}\text{ kN (c)}}$ (25.46 kN)

$15\sqrt{5}$

$\uparrow \Sigma F_y = A_y + F_{AB}\sin\theta + F_{BF} + F_{CF}\sin\alpha - 12 = 0$

$F_{BF} = -30 - \left(-15\sqrt{5}\cdot\frac{2}{\sqrt{5}}\right) - \left(-18\sqrt{2}\times\frac{1}{\sqrt{2}}\right) + 12$

$F_{BF} = 18 + 12$

$F_{BF} = \underline{30\text{ kN (T)}}$

(d) cross sectional area of CD

$F_{CD} = 6.71$ kN

$\sigma_{all} \geq \frac{F_{CD}}{area} \Rightarrow area \geq \frac{6.71\times10^3\text{ N}}{150\text{ N/mm}^2} = \underline{44.73\text{ mm}^2}$

(e) change in length of CD

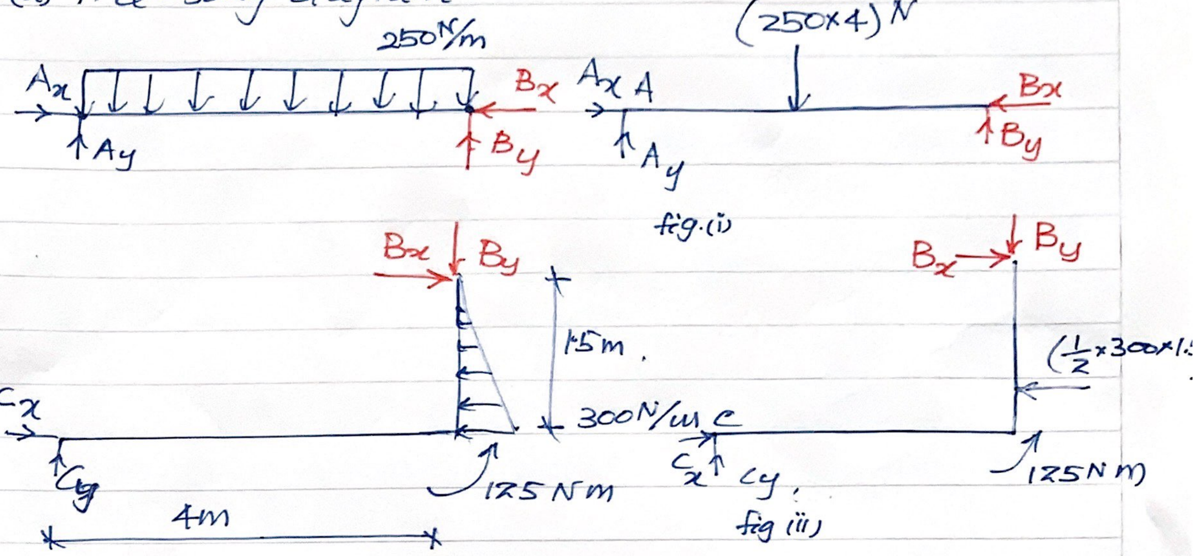
$\frac{F}{A} = E\frac{\Delta}{L}$ $\quad \Delta = \frac{FL}{AE} = \frac{(6.71\times10^3)\text{N}\cdot 2\sqrt{5}\times10^3}{44.73\text{ mm}^2\times 205\times10^3\text{ N/mm}^2} = \underline{3.27\text{ mm}}$

No, change in length is not equal to the vertical displacement of point D.

(f) If the selfweight of the truss members are significantly large,

- we have to consider member weights into consideration.
- we distribute meber weights to joint are a joint force.
- Then calculate memberforce using any method.

(Q2) (a) Free-body diagram



(b) Support reactions

taking moment at A → fig(i)

$\circlearrowleft M_A = 4B_y - (250 \times 4 \times 2) = 0 \qquad B_y = \underline{500\,N}$

taking moment at C (fig ii)

$\circlearrowleft \quad -4B_y - 1.5B_x + \left(\frac{1}{2} \cdot 300 \times 1.5\right) \cdot \frac{1.5}{3} + 125 = 0$

$+1.5B_x = \left(\frac{1}{2} \times 300 \times 1.5 \times \frac{1.5}{3}\right) + 125 - 4 \times 500$

$B_x = \underline{\underline{-1175\,N}}$

Fig (i)

$\rightarrow \quad A_x + B_x = 0 \qquad A_x = -(-1175) = \underline{1175\,N}$

$\uparrow \quad A_y + B_y - 1000 = 0 \qquad A_y = 1000 - 500 = \underline{\underline{500\,N}}$

Fig (ii)

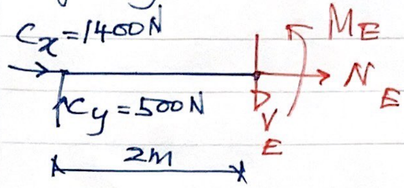
$\uparrow \quad C_y - B_y = 0 \qquad C_y = \underline{\underline{500\,N}}$

$\rightarrow \quad C_x + B_x - \left(\frac{1}{2} \cdot 300 \times 1.5\right) = 0 \qquad C_x = \left(\frac{1}{2} \times 300 \times 1.5\right) - (-1175)$

$C_x = \underline{1400\,N}$

(c) Internal forces

imaginary cut at E



considering equilibrium

$\uparrow \; C_y - V_E = 0$

$V_E = \underline{500\,N}$

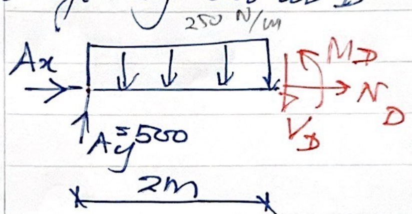
$\rightarrow N_E + C_x = 0$

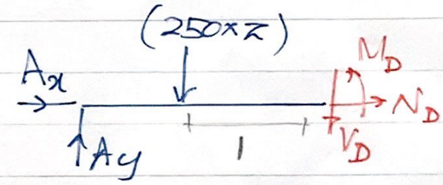
$N_E = \underline{-1400N}$

$\curvearrowleft \; M_E - 500N \cdot 2 = 0$

$M_E = \underline{1000\,Nm}$

imaginary cut at D





$\rightarrow A_x + N_D = 0 \qquad N_D = \underline{+1175N}$

$\uparrow A_y - V_D - 500 = 0 \quad -V_D = 500 - 500 = 0$

$V_D = \underline{0}$

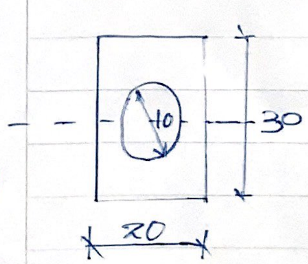
$\curvearrowleft \quad M_D + (250 \times 2 \times 1) - 500 \times 2 = 0$

$M_D = 1000 - 500 = \underline{500}\,Nm$

(d) cross sectional area

$= (30 \times 20) - (\pi \times 5^2) = \underline{521.46\,mm^2}$

moment of inertia



$I = \left(\frac{1}{12} \cdot 20 \cdot 30^3\right) - \left(\frac{1}{4}\pi 5^4\right)$

$= \underline{44509.13\,mm^4}$

/ /

(e) Maximum normal stress

σ = Normal stress due to axial force (σ_N) + Normal stress due to bending (σ_m)

$$\sigma = \frac{P}{A} + \left(\frac{-My}{I}\right)$$

at D

$$\frac{1175N}{521.46\,mm^2} + \left(\frac{500 \times 10^3 \times 15}{44509.13}\right) = \underline{\underline{170.758\ MPa}}$$

at E

$$= \frac{-1400N}{521.46\,mm^2} + \left(\frac{-1000 \times 10^3\,Nmm \times 15mm}{44509.13\,mm^4}\right) = \underline{\underline{-339.69\ Mpa}}$$

Maximum shear stress

at D

$$\tau_{max} = \frac{VQ}{It} = \underline{\underline{0}}$$

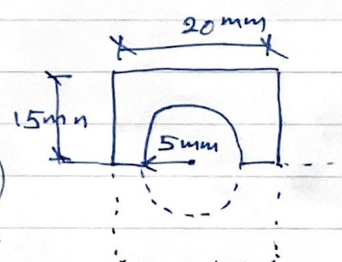
at E

$V = 500N$

$t = 10mm$

$I = 44509.13mm^4$

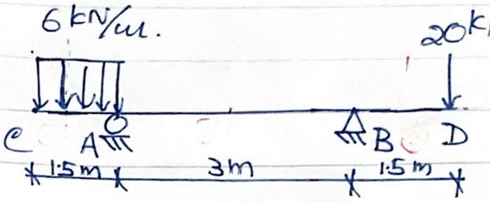
$$Q = \Sigma A'\bar{y}' = (20 \times 15) \cdot \frac{15}{2} - \left(\frac{\pi \times 5^2}{2} \cdot \frac{4 \times 5}{3\pi}\right)$$

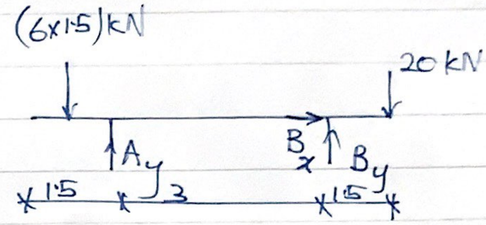


$$= 2166.667\ mm^3$$

$$\tau_{max} = \frac{500N \times 2166.67mm^3}{44509.13mm^4 \times 10mm} = \underline{\underline{2.43\ Mpa}}$$

(Q 3) (a)





considering equilibrium.

→ $B_x = 0$

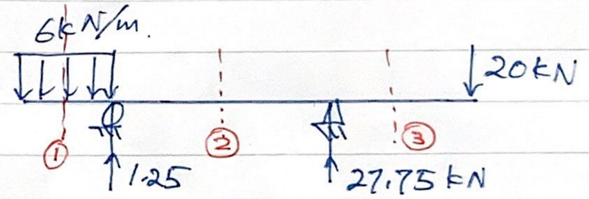
↑ $A_y + B_y = 20 + (6\times1.5) = 29$

B: $(-20\times1.5) - (A_y\,3) + \left(9\times 3 + \frac{1.5}{2}\right) = 0$

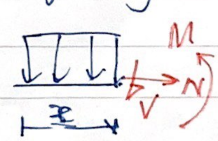
$A_y = 1.25$ kN

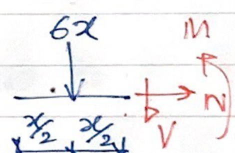
$B_y = 29 - 1.25 = 27.75$ kN

(b)



imaginary cut 1





x	V	M
0	0	0
1.5	-9	-6.75

considering equilibrium

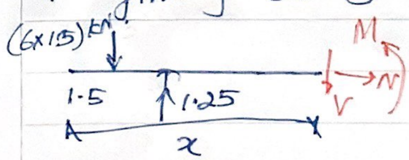
↓ $V + 6x = 0$

$V = -6x$

↺ $M + 6x\cdot\frac{x}{2} = 0$

$M = -3x^2$

imaginary cut 2



↓ $V + (6\times1.5) - 1.25 = 0$

$V = -7.75$

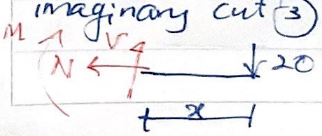
↺ $M + \left(9\left(x - \frac{1.5}{2}\right)\right) - 1.25(x - 1.5) = 0$

$M + 9x - 6.75 - 1.25x + 1.875 = 0$

$M = -7.75x + 4.875$

x	V	M
1.5	-7.75	-6.75
4.5	-7.75	-30

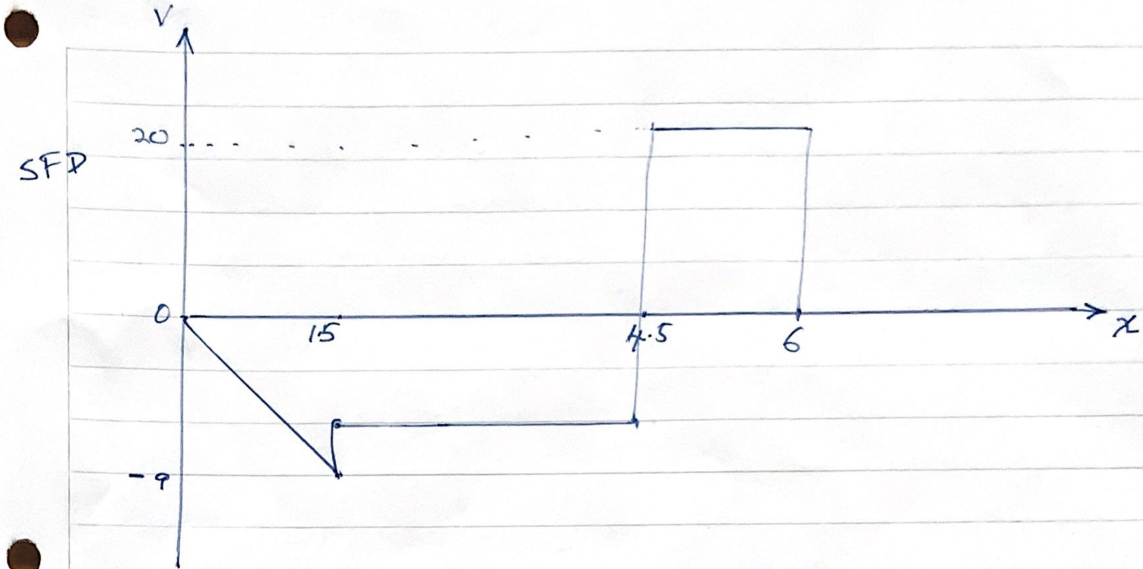
imaginary cut 3

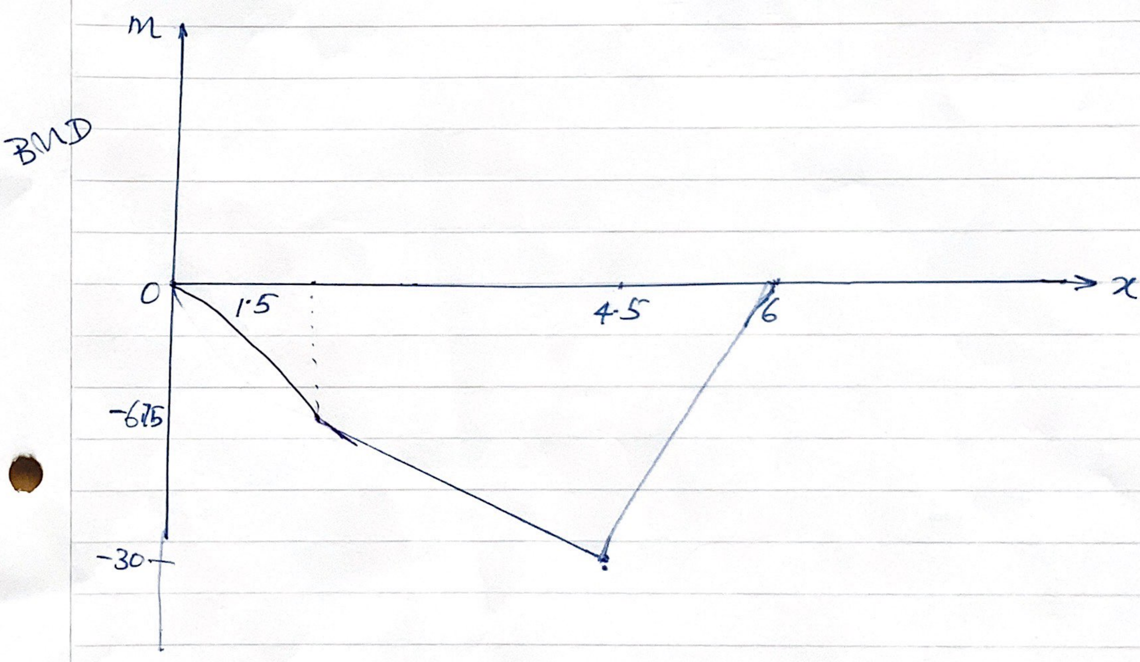


↑ $V - 20 = 0$ $V = 20$

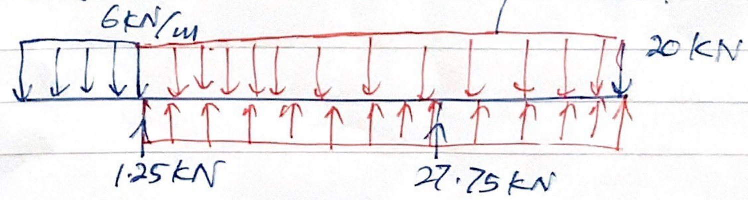
↺ $M + 20x = 0$ $M = 20x$

x	V	M
0	20	0
1.5	20	30





(d) Deflection at the end point D.



$$M = 1.25\langle x-1.5\rangle^{1} - \frac{6}{2}\langle x-0\rangle^{2} + \frac{6}{2}\langle x-1.5\rangle^{2} + 27.75\langle x-4.5\rangle^{1}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 1.25\langle x-1.5\rangle^1 - 3x^2 + 3\langle x-1.5\rangle^2 + 27.75\langle x-4.5\rangle^1$$

$$EI\frac{dv}{dx} = \frac{1.25}{2}\langle x-1.5\rangle^2 - x^3 + \frac{3}{3}\langle x-1.5\rangle^3 + \frac{27.75}{2}\langle x-4.5\rangle^2 + C_1$$

$$EI\,v = \frac{1.25}{6}\langle x-1.5\rangle^3 - \frac{x^4}{4} + \frac{3}{12}\langle x-1.5\rangle^4 + \frac{27.75}{6}\langle x-4.5\rangle^3 + C_1x + C_2$$

Using boundary conditions.

$x=1.5 \quad v=0$

$x=4.5 \quad v=0$

from eq.(2)

$$EI\times 0 = -\frac{1.5^4}{4} + C_1 1.5 + C_2 \qquad C_1 1.5 + C_2 = 1.266 \text{ — ①}$$

$$0 = \frac{1.25}{6}(4.5-1.5)^3 - \frac{4.5^4}{4} + \frac{3}{12}(4.5-1.5)^4 + 4.5C_1 + C_2$$

$$4.5C_1 + C_2 = 76.64 \text{ — ②} \longrightarrow C_2 = 76.64 - 4.5C_1$$

① & ② ⇒ $C_1 = 25.12$

$C_2 = -36.4$

Then

$$v = \frac{1}{EI}\left\{\frac{1.25}{6}\langle x-1.5\rangle^3 - \frac{x^4}{4} + \frac{3}{12}\langle x-1.5\rangle^4 + \frac{27.75}{6}\langle x-4.5\rangle^3 + 25.12x - 36.4\right.$$

At D, $x = 6m$

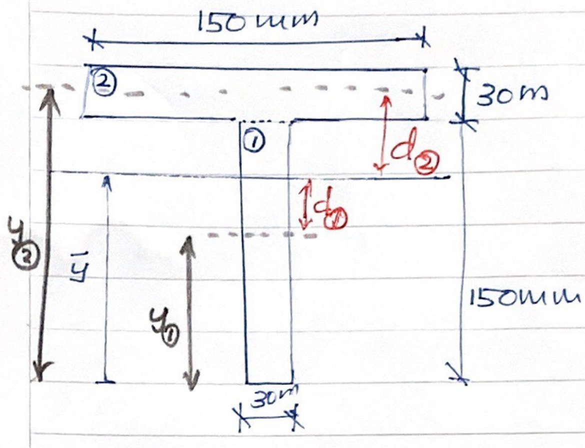
Deflection

$$v = \frac{1}{EI}\left\{\frac{1.25}{6}(6-1.5)^3 - \frac{6^4}{4} + \frac{3}{12}\langle 6-1.5\rangle^4 + \frac{27.75}{6}(6-4.5)^3 + (25.12\times 6) - 36.4\right.$$

$$v = \frac{-72.570625}{210\times10^9\times 2.7\times10^{-3}} = 0.0128m \qquad \underline{\underline{v = 12.8\ mm}}$$

(e) magnitude of the deflection at D will reduce. Due to the change of deflected shape

(c) moment of inertia.



$$\bar{y} = \frac{A_{①}\tilde{y}_{①} + A_{②}\bar{y}_{②}}{A_{①} + A_{②}}$$

$$\bar{y} = \frac{\left(150 \times 30 \times \frac{150}{2}\right) + \left(150 \times 30 \times \left(150 + \frac{30}{2}\right)\right)}{(150 \times 30) + (150 \times 30)}$$

$= 120\text{mm}$.

$$I = \left[\left(\frac{1}{12} \cdot 30 \times 150^3\right) + (150 \times 30) \times (120 - 75)^2\right] +$$

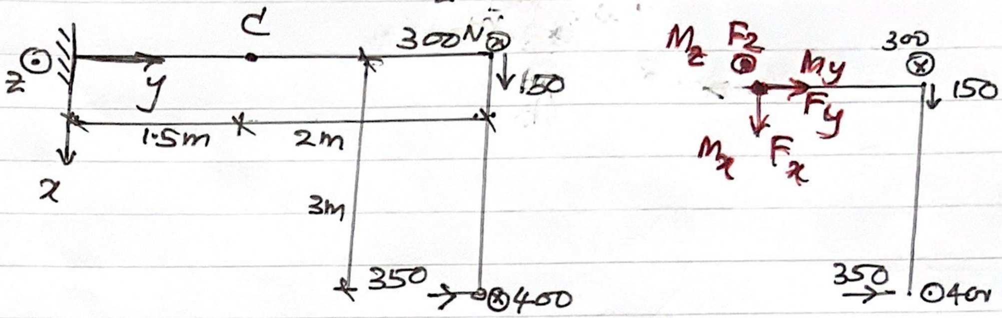
$$\left[\left(\frac{1}{12} \cdot 150 \times 30^3\right) + (150 \times 30)\ (165 - 120)^2\right]$$

$= \underline{27000000 \text{ mm}^4}$

(Q4)

(a)

$\vec{F}_2 = 150\hat{i} - 300\hat{k}$



$\vec{F}_1 = 350\hat{j} - 400\hat{k}$

$\Sigma F_x = F_x + 150 = 0$ $\qquad F_x = \underline{-150\,N}$

$\Sigma F_y = F_y + 350 = 0$ $\qquad F_y = \underline{-350\,N}$

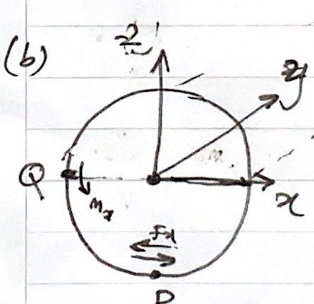
$\Sigma F_z = F_z - 300 - 400 = 0$ $\qquad F_z = \underline{700\,N}$

$\Sigma M_x = M_x - (300 \times 2) - (400 \times 2) = 0$ $\qquad M_x = \underline{1400\,Nm}$

$\Sigma M_y = M_y + (400 \times 3) = 0$ $\qquad M_y = \underline{-1200\,Nm}$

$\Sigma M_z = M_z + (350 \times 3) - (150 \times 2) = 0$ $\qquad M_z = \underline{-750\,Nm}$

(b)



$\sigma_P = \frac{F_y}{A} - \frac{M_x r}{I}$

$= \frac{350}{\pi r^2} - \frac{1400 \times 10^6 \cdot r}{\frac{1}{4}\pi r^4} = \frac{350}{\pi r^2} - \frac{1400 \times 10^3}{\frac{\pi r^3}{4}}$

diameter(d) = 60mm $\qquad \sigma_P = \underline{\underline{-65.866\ N/mm^2}}$

$\tau_P = \frac{Tr}{J} - \frac{|F_x| Q}{I(2r)}$

$= \frac{1200 \times 0.03}{1.2723 \times 10^{6}} - \frac{150 \times 1.8 \times 10^{-5}}{6.3617 \times 10^{6}\ 0.06}$

$= \underline{\underline{28.22\ N/mm^2}}$

$J = \frac{\pi}{2} r^4 = 1.272 \times 10^{-6}\,m^4$

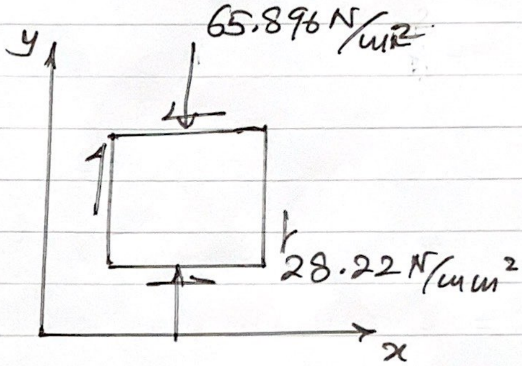
$I = \frac{\pi}{4} r^4 = 6.36 \times 10^{-7}\,m^4$

$Q = \frac{\pi R^2}{2} \cdot \frac{4R}{3\pi} = 1.8 \times 10^{-5}\,m^3$

$$\sigma_q = \frac{F_y}{A} - \frac{M_z r}{I} = \frac{350}{\pi r^2} - \frac{750 \times 10^3 r}{\frac{1}{4}\pi r^4} = \underline{-35.244 \ N/mm^2}$$

$$\tau_q = \frac{Tr}{J} - \frac{|F_z| Q}{I(2r)} = \underline{27.96 \ N/mm^2}$$

At P



$\sigma_x = 0$

$\sigma_y = -65.896 \ N/mm^2$

$\tau_{xy} = -28.22 \ N/mm^2$

(c) $$\sigma_{1,2} = \frac{\sigma_1 + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = -\frac{65.896}{2} \pm \sqrt{\left(\frac{65.896}{2}\right)^2 + (28.22)^2}$$

$\underline{\sigma_1 = 10.43 \ N/mm^2}$ $\qquad$ $\underline{\sigma_2 = -76.33 \ N/mm^2}$

$$2\theta_p = \tan^{-1}\left(\frac{\tau}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}\right) = \tan^{-1}\left(\frac{28.136}{\left(\frac{65.896}{2}\right)}\right) = 40.58$$

$$\theta_p = \underline{20.29^\circ}$$

(d) Unstable structure. It becomes a mechanism.