LØSNINGSFORSLAG H20

1

a) $z = 1 + 4i, \quad w = -3 + 4i$

$$zw = (1+4i)(-3+4i) = -3 + 4i - 12i + 4^2 i^2$$

$$= -3 - 8i + 16 \cdot (-1) = \underline{\underline{-19 - 8i}}$$

$$\frac{w}{z} = \frac{-3+4i}{1+4i} = \frac{(-3+4i)(1-4i)}{(1+4i)(1-4i)} = \frac{-3+4i+12i-4^2i^2}{1^2+4^2}$$

$$= \frac{-3+16i+16}{17} = \underline{\underline{\frac{13}{17} + \frac{16i}{17}}}$$

$$\bar{z}\bar{w} = \overline{zw} = \overline{-19-8i} = \underline{\underline{-19+8i}}$$

b) $-64 = 64 \cdot (-1) = \underline{64 e^{\pi i}}$

4. RØTTER:

$$w_1 = 64^{\frac{1}{4}} e^{\frac{\pi i}{4}} = 2^{6 \cdot \frac{1}{4}} e^{\frac{\pi i}{4}} = 2^{\frac{3}{2}} e^{\frac{\pi i}{4}}$$

$$= 2\sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right) = 2\sqrt{2}\left(\tfrac{1}{2}\sqrt{2} + i\tfrac{1}{2}\sqrt{2}\right)$$

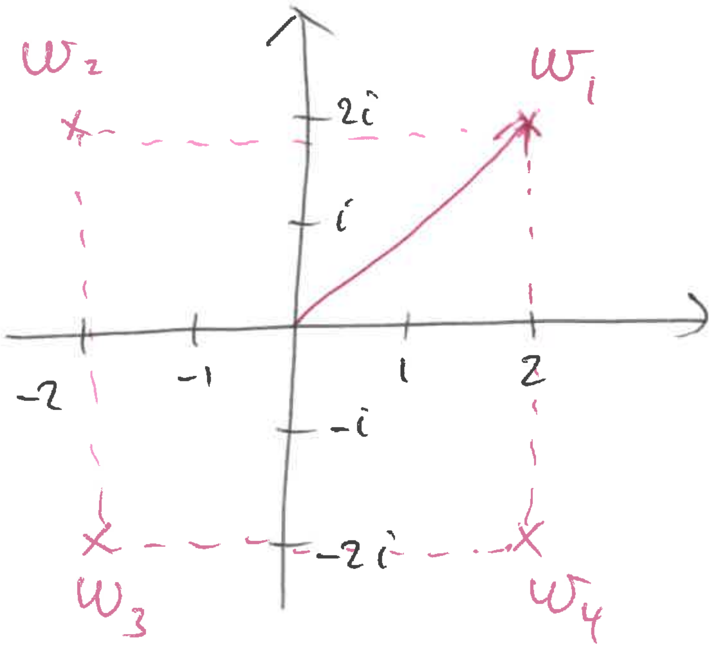
$$= \underline{\underline{2 + 2i}}$$

$$w_2 = 64^{\frac{1}{4}} e^{\frac{\pi i}{4} + \frac{2\pi}{4} i} = 2\sqrt{2}\, e^{\frac{3\pi}{4} i} = 2\sqrt{2}\left(-\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\sqrt{2}\, i\right)$$

$$= \underline{\underline{-2 + 2i}}$$

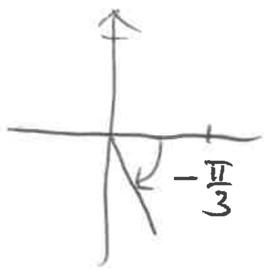
$$w_3 = 64^{\frac{1}{4}} e^{\frac{\pi i}{4} + \frac{4\pi}{4} i} = \underline{\underline{-2 - 2i}}$$

$$w_4 = 64^{\frac{1}{4}} e^{\frac{\pi i}{4} + \frac{6\pi}{4} i} = \underline{\underline{2 - 2i}}$$



c) $z = 1 - \sqrt{3}\, i = 2 \cdot \left(\tfrac{1}{2} - \tfrac{1}{2}\sqrt{3}\, i\right)$

$$\underline{\underline{= 2 e^{-\frac{\pi}{3} i}}}$$



$z^n = \left(2 e^{-\frac{\pi}{3} i}\right)^n = 2^n e^{-\frac{\pi}{3} \cdot n}$ ER REELT

TALL NÅR $\frac{\pi}{3} n = k \cdot \pi$. $k \in \mathbb{Z}$

$\Rightarrow n = 3k$. SIDEN n POSITIVT $\Rightarrow$ $\underline{\underline{n = 3K}}$, $K \in \mathbb{N}$

② a) $\int (3\cancel{x}^{-1/2} + e^{2\pi x})\,dx$

$= 3\cdot 2x^{1/2} + \frac{1}{2\pi}e^{2\pi x} + C$

$= \underline{\underline{6\sqrt{x} + \frac{1}{2\pi}e^{2\pi x} + C}}$

b) $\int \frac{\sqrt{\tan^{-1}x}}{1+x^2}\,dx$

$\boxed{\begin{aligned} u &= \tan^{-1}x \\ du &= \frac{1}{1+x^2}\,dx \end{aligned}}$

$= \int \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2} + C = \underline{\underline{\frac{2}{3}(\tan^{-1}x)^{3/2} + C}}$

c) $\int \frac{5x+9}{x(x+3)^2}\,dx$

DELBRØKOPPSPALTNING:

$\frac{5x+9}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$= \frac{A(x+3)^2 + B\cdot x(x+3) + Cx}{x(x+3)^2}$

$$5x+9 = A(x+3)^2 + Bx(x+3) + Cx$$

$\underline{x=0:}$ $\quad 9 = A \cdot 3^2 \Rightarrow \underline{A=1}$

$\underline{x=-3:}$ $\quad -15+9 = A \cdot 0 + B \cdot 0 + C \cdot (-3)$

$$-6 = -3C \Rightarrow \underline{C=2}$$

$\underline{x=1:}$ $\quad 5+9 = A \cdot 4^2 + B \cdot 1 \cdot 4 + C$

$$14 = 1 \cdot 16 + 4B + 2$$

$$14 = 18 + 4B \Rightarrow \underline{B=-1}$$

$$\int \frac{5x+9}{x(x+3)^2}\,dx = \int \left(\frac{1}{x} - \frac{1}{x+3} + \frac{2}{(x+3)^2} \right) dx$$

$$= \underline{\underline{\ln|x| - \ln|x+3| - \frac{2}{x+3} + C}}$$

③ $f(x) = \frac{\sqrt{x}}{\sqrt{1+x^2}}\ , \quad x \geq 0$

a) $f'(x) = \frac{\frac{1}{2\sqrt{x}} \cdot \sqrt{1+x^2} - \sqrt{x}\frac{\cdot 2x}{2\sqrt{1+x^2}}}{\left(\sqrt{1+x^2}\right)^2}$ $\cdot \frac{\sqrt{x}\sqrt{1+x^2}}{\sqrt{x}\sqrt{1+x^2}}$

$= \frac{1+x^2 - 2x^2}{2\sqrt{x}\sqrt{1+x^2}(1+x^2)} = \underline{\frac{1-x^2}{2\sqrt{x}\sqrt{1+x^2}(1+x^2)}}$

$f'(x) = 0$ NÅR $1 - x^2 = 0$

$\Rightarrow \underline{x = \pm 1}$

SIDEN $x \geq 0 \Rightarrow \underline{x = 1} \Rightarrow f(1) = \frac{1}{\sqrt{2}} = \underline{\frac{1}{2}\sqrt{2}}$

ENDEPUNKT: $\underline{x = 0} \Rightarrow f(0) = \underline{0}$

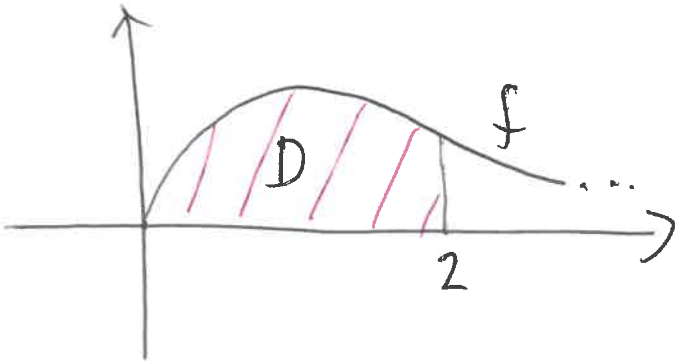
NÅR $x \to \infty$:

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{1+x^2}} = \sqrt{\lim_{x\to\infty}\frac{x}{1+x^2}} = \underline{0}$

MEN $f(x) > 0$ FOR $x > 0$.

SÅ <u>MAXIMUM</u> FOR $x=1$: $f_{max} = \underline{\underline{\frac{1}{2}\sqrt{2}}}$ GLOBALT.

MINIMUM FOR $x=0$: $f_{min} = \underline{\underline{0}}$ GLOBALT b)



$$V = \pi \int_0^2 [f(x)]^2 \, dx = \pi \int_0^2 \frac{x}{1+x^2} \, dx$$

$$\boxed{\begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array}}$$

$$= \pi \int_{x=0}^{x=2} \frac{\frac{1}{2}}{u} \, du = \pi \ln u \Big|_{x=0}^{x=2} = \frac{\pi}{2} \ln(1+x^2) \Big|_0^2$$

$$= \frac{\pi}{2} (\ln 5 - \ln 1) = \underline{\underline{\frac{\pi}{2} \ln 5}}$$

4 a)

$$\begin{cases} y' - \frac{2}{x}y = x^2 \ln x \\ y(1) = 2 \end{cases}$$

$$y' - \frac{2}{x}y = x^2 \ln x$$

$$\mu = \int \left(-\frac{2}{x}\right) dx = -2\ln x$$

INT. FAKTOR: $e^{\mu} = e^{-2\ln x} = \left(e^{\ln x}\right)^{-2}$

$$= \underline{x^{-2}}$$

$$y' - \frac{2}{x}y = x^2 \ln x \quad \Big| \cdot x^{-2}$$

$$\left(yx^{-2}\right)' = \cancel{x^2} \ln x \cdot \cancel{x^{-2}} = \ln x$$

$$yx^{-2} = \int \ln x \, dx = x\ln x - x + C$$

$$y = \underline{x^3 \ln x - x^3 + C \cdot x^2}$$

$$y(1) = 1 \cdot \ln 1 - 1 + C \cdot 1 = 2 \Rightarrow C = 3$$

LØSNING: $y(x) = \underline{\underline{x^3 \ln x - x^3 + 3x^2}}$

b) $y'' + 3y' - 10y = 6e^{2x}$

HOM. LIGN:

$$y'' + 3y' - 10y = 0$$

$$r^2 + 3r - 10 = 0$$

$$\underline{r_1 = -5 \qquad r_2 = 2}$$

$$\underline{y_h = Ae^{-5x} + Be^{2x}}$$

PARTIKULÆR LØSN:

SIDEN $6e^{2x}$ ER EN HOM. LØSNING PRØVER VI:

$$y_p = a \cdot xe^{2x}$$

$$y_p' = ae^{2x} + 2axe^{2x}$$

$$y_p'' = 2ae^{2x} + 2ae^{2x} + 4axe^{2x}$$

$$y_p'' + 3y_p' - 10y_p = 4ae^{2x} + \underline{4axe^{2x}} + 3ae^{2x} + \underline{6axe^{2x}} \quad = 0$$

$$-\underline{10axe^{2x}}$$

$$= \underline{7ae^{2x}} = \underline{6e^{2x}}$$

$$\Rightarrow a = \frac{6}{7}$$

LØSNING: $y = y_h + y_p = Ae^{-5x} + Be^{2x} + \frac{6}{7}xe^{2x}$

c) $\lim_{x \to 0} \frac{1-\cos(bx)}{x^2} \overset{L'H}{=} \lim_{x \to 0} \frac{b\sin(bx)}{2x}$

$\overset{L'H}{=} \lim_{x \to 0} \frac{b^2\cos(bx)}{2} = \frac{b^2}{2}$

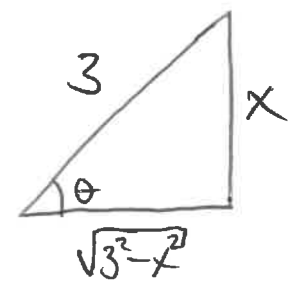
SÅ:

$$\frac{b^2}{2} = 2020$$

$$b^2 = 2 \cdot 2020 = 4 \cdot 1010$$

$$b = \pm\sqrt{4 \cdot 1010} = \pm 2\sqrt{1010}$$

d) $\int \frac{1}{(9-x^2)^{3/2}}\,dx$



$= \int \frac{3\cos\theta\,d\theta}{(3\cos\theta)^3}$

$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{1}{3}\sqrt{9-x^2}$

$= \int \frac{1}{9}\frac{1}{\cos^2\theta}\,d\theta$

$dx = 3\cos\theta\,d\theta$

4d) FORTS

$$= \frac{1}{9}\tan\theta + C = \frac{1}{9}\frac{x}{\sqrt{9-x^2}} + C$$

5 a) $2x^3 + x^2y + 2y^3 = 10$

FOR $(x,y) = (2,-1)$:

VS: $2\cdot 2^3 + 2^2\cdot(-1) + 2\cdot(-1)^3 = 16 - 4 - 2 = 10$

HS: 10

DVS. $P(2,-1)$ LIGGER PÅ KURVA;

SKJÆRING AV:

X-AKSEN: $(x,0)$

$$\Rightarrow 2x^3 + x^2\cdot 0 + 2\cdot 0^2 = 10$$

$$2x^3 = 10$$

$$x = \sqrt[3]{5}$$

$$(\sqrt[3]{5}, 0)$$

y-AKSEN: $(0,y) \Rightarrow 2y^3 = 10 \Rightarrow y = \sqrt[3]{5}$

$$(0, \sqrt[3]{5})$$

b) $\frac{d}{dx}(2x^3+x^2y+2y^3)=\frac{d}{dx}(10)$

$$6x^2+2xy+x^2y'+6y^2y'=0$$

$$(x^2+6y^2)y'=-6x^2-2xy$$

$$y'=-\frac{2x(3x+y)}{x^2+6y^2}$$

c) FOR $P(2,-1)$

$$y'\Big|_P=-\frac{2\cdot2(3\cdot2-1)}{2^2+6\cdot1^2}=-\frac{4\cdot5}{10}=-2$$

TANGENT: $a=-2$

$$y+1=-2(x-2)=-2x+4$$

$$y=-2x+3$$

NORMAL:

$$y+1=\frac{1}{2}(x-2)=\frac{1}{2}x-1$$

$$y=\frac{1}{2}x-2$$

⑥ $N'(t) = kN(A-N)$

$A = 700$

$N'(0) = 1$

a) $N(0) = 2$. FOR $t=0$:

$$N'(0) = kN(0)\cdot(700 - N(0))$$

$$\underline{1} = k\cdot 2\cdot(700-2) = K\cdot 2\cdot 698$$

$$\Rightarrow k = \frac{1}{2\cdot 698} = \underline{\frac{1}{1396}}$$

$$\frac{dN}{dt} = kN(A-N)$$ Separabel

$$\frac{dN}{N(A-N)} = k\,dt$$

$$\int \frac{dN}{N(A-N)} = \int k\,dt = kt + C$$

$$\int \frac{dN}{N(A-N)} = \int \left(\frac{a}{N} + \frac{b}{A-N}\right) dN$$

HER BLIR $a = b = \frac{1}{A}$

$$\int \frac{dN}{N(A-N)} = \frac{1}{A} \int \left(\frac{1}{N} + \frac{1}{A-N} \right) dN$$

$$= \frac{1}{A} \left[\ln N - \ln(A-N) \right] + C_2$$

$$= \frac{1}{A} \ln \frac{N}{A-N} + C_2$$

$$\frac{1}{A} \ln \frac{N}{A-N} + C_2 = kt + C$$

KAN SETTE $C_2 = 0$.

$$\ln \frac{N}{A-N} = A(kt + C)$$

$$\frac{N}{A-N} = e^{Akt + AC} = e^{Akt} \cdot D \quad , \quad \underline{D \equiv e^{AC}}$$

SETTER $N(0) = 2$

$$\frac{2}{700-2} = e^{0} \cdot D \Rightarrow D = \frac{2}{698} = \underline{\frac{1}{349}}$$

LØSER $\frac{N}{A-N} = D \cdot e^{Akt}$ M.H.P. N

$$N(t) = \frac{Ae^{Akt}}{349 + e^{Akt}} = \frac{A}{1 + 349e^{-Akt}}$$

b) NÅR ER

$N(t) = 350 = \frac{A}{2}$ (50%).

$$\Rightarrow \frac{A}{1+349e^{-Akt}} = \frac{A}{2}$$

$$\Rightarrow 1 + 349e^{-Akt} = 2$$

$$349e^{-Akt} = 1 \quad | \cdot e^{Akt}$$

$$349 = e^{Akt}$$

$$Akt = \ln 349$$

$$t = \frac{1}{Ak} \ln 349 = \frac{1396}{700} \ln 349$$

$$\simeq \underline{11{,}5}$$

DVS $\underline{11{,}5 \text{ DAGER}}$

NÅR $t \to \infty$, SÅ GÅR

$e^{-Akt} \to 0$. DERMED:

$$N(t) = \frac{A}{1+349e^{-Akt}} \to \underline{\underline{A = 700}}$$

SÅ HELE ALVEBEFOLKNINGEN BLIR SMITTA...