Høsten 2020

FYS100 Fysikk: Exam/Eksamen

You **must** put your candidate number on every sheet.

There are 4 questions. You need to answer all 4 questions for a full score.

The standard formula sheet for FYS100 Fysikk is part of this question sheet.

Standard approved calculators are allowed.

Don't panic! Draw a diagram where relevant. State clearly the relevant physics.

The questions are also attached in Norwegian.

Good luck!

Du **må** legge kandidatnummeret ditt på hvert ark.

Det er 4 spørsmål. Du må svare på alle de fire spørsmålene for en full score.

Standardformelarket for FYS100 Fysikk er en del av dette spørsmålet.

Standard godkjente kalkulatorer er tillatt.

Ingen panikk! Tegn et diagram der det er relevant. Angi tydelig hvilken fysikk som er relevant.

Spørsmålene er også vedlagt på engelsk.

Lykke til!

Problem 1: Bobsleigh

a) A bobsleigh of mass 210kg is pushed from rest with constant acceleration by two runners with a total mass of 180kg. The runners push the bobsleigh a distance 48m along a horizontal track in a time 2.8s. Take the coefficient of kinetic friction between the ice and the bobsleigh to be 0.02. What is the initial force exerted on the bobsleigh by the two runners once the bobsleigh is moving?

Solution: Use Newton's second law $F = ma$. There are two horizontal forces acting on the bob, the push from the runners and the friction from the ice. The friction from ice is given by

$$f_k = \mu_k N = u_k mg = 0.02 * 210 * 9.8 = 41.16 = 41\text{N}(2\text{sig.fig.}) \tag{1}$$

The acceleration of the bob is given by the kinematic equation $s = ut + \frac{1}{2}at^2$.

$$a = \frac{2s}{t^2} = \frac{2 * 48}{2.8^2} = 12.24 = 12ms^{-2}(2\text{sig.fig.}) \tag{2}$$

The push from the runners needs to overcome the friction and accelerate the bob so

$$F_{push} = ma + f_k = 210 * 12 + 41 = 2561 = 2600\text{N}(2\text{sig.fig.}) \tag{3}$$

b) The runners then quickly jump into the bobsleigh and it moves down a straight track of length 52m and slope 11 degrees to the horizontal. Draw a clear diagram of the situation, indicating the direction of the relevant forces on the bobsleigh. Using the available information, calculate the speed of the bobsleigh at the bottom of the slope. List any approximations you make in calculating this.

Solution: Use Newton's second law $F = ma$. Block on a slope. Resolve forces perpendicularly to the slope

$$N - mg\cos\theta = 0 \tag{4}$$

Resolve forces parallel to the slope

$$mg\sin\theta - f_k = ma \tag{5}$$

Solve these two equations for the acceleration a by eliminating the normal force using $f_k = \mu_k N$. This gives

$$a = g\sin\theta - \mu_k g\cos\theta = 9.8(\sin 11 - 0.02 * \cos 11) = 1.6775 = 1.7\text{ms}^{-2}(2 sig.fig.) \tag{6}$$

The final speed is obtained by the kinematic relation $v_f^2 = v_i^2 + 2as$. The initial velocity v_i can be approximated from the information in part (a) as $v_i = \sqrt{2 * 12 * 48} = 33.9 = 34\text{ms}^{-1}$. then the final speed at the bottom of the slope is

$$v_f = \sqrt{(34)^2 + 2 * 1.7 * 52} = 36.45 = 36\text{ms}^{-1}(2sig.fig.) \tag{7}$$

In doing this calculation, we have assumed that the initial velocity at the top of the slope is given by the information in part (a). The runners jump instantaneoulsy into the bob and no speed change occurs when the slope begins. We have also ignored any air resistance and assumed that the slope is smooth and everywhere has a slope of 11 degrees.

c) After reaching the bottom of the slope, the bobsleigh enters a bend on level ground. The radius of this bend is 36m. What angle to the vertical would the bobsleigh have to tilt by (by riding up the side of a curved track) in order to make such a bend without additional steering?

Solution: To take the turn, the bob needs a centripetal acceleration of $a_c = \frac{v^2}{r} = \frac{36^2}{36} = 36\text{ms}^{-2}$. Without additional steering this centripetal acceleration must be provided by the horizontal component of the normal force on a slope of angle ϕ to the vertical. So we use Newton's second law and resolve forces. The is no vertical acceleration so forces resolved vertically give

$$N \sin\phi - mg = 0 \tag{8}$$

Forces resolved horizontally give

$$N \cos\phi = ma_c \tag{9}$$

Solving these two equations by eliminating N gives $\phi = \tan^{-1}\left(\frac{g}{a_c}\right) = \tan^{-1}\left(\frac{9.8}{36}\right) = 15.23 = 15$degrees (2 sig. fig.).

d) Describe in words why it might be an advantage to have heavier runners in order to get down the slope faster.

Solution: In this problem the masses have cancelled out. This is because we have only considered forces due to gravity, normal forces and frictional forces. If we include additionally air resistance forces, the mass will not cancel out and we will find that having a heavier mass makes the bob go faster down the slope. This is similar to why a bowling ball falls faster than a feather in the presence of air

resistance. The air resistance component of the acceleration will be suppressed by a factor 1/m.

Problem 2: Washing machine

a) A washing machine drum has a radius of 25cm. The drum is rotated from rest by an electrical motor until it is rotating 11 times per second. The drum takes 0.50 seconds to be sped up by the motor. What constant torque must the electrical motor apply if the washing machine contains wet clothes weighing 4.0kg and we assume that the clothes distribute themselves uniformly around the rim of the washing machine drum? The moment of inertia of a thin cylindrical shell is $I = MR^2$.

Solution: The torque is given by the angular equivalent of Newton's second law $\tau = I\alpha$. We can find the angular acceleration α from the given data by using a kinematic relation $\alpha = \omega_f/t$. The final angular velocity is given by $\omega_f = 2\pi f = 2 * 3.14159 * 11 = 69.1\text{s}^{-1}$. Putting it all together

$$\tau = I\alpha = \frac{MR^2 2\pi f}{t} = \frac{4 * 0.25^2 * 2 * 3.14159 * 11}{0.5} = 35\text{Nm} \tag{10}$$

b) Assuming the electrical motors apply a constant torque, through what angle does the washing machine drum rotate in the 0.50 seconds it takes to be sped up by the motor?

Solution: This can be obtained from the kinematic relation $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} * 140 * \left(\frac{1}{2}\right)^2 = 17.5$ radians. This is 2.8 full rotations or 1000 degrees to 2 sig. fig.

c) If the washing machine contains instead 8kg of wet clothes, explain what would happen differently to the case with 4kg of wet clothes and why. Would it take longer or shorter to spin up the drum? Would the electrical motor use more or less energy? Assume that the electrical motor will apply the same torque and the drum is sped up to the same rate of rotation.

Solution: If the clothes are twice as heavy, then the moment of inertia will be doubled. If the torque is the same then the angular acceleration will be halved. It will take twice as long to spin up to the same frequency. The amount of rotational energy added by the electric motor will be doubled. The angle rotated through will be doubled and the work done by the electric motor will be doubled.

d) During the rapid rotation of the washing machine, water is removed from the clothes through small holes in the walls of the drum. Explain in terms of Newton's laws of motion and conservation laws why water can be removed in this way.

Solution: By Newton's first law, both the clothes and the water in the clothes will follow straight lines unless acted on by a force. This is conservation of momentum. The wall on the edge of the drum exerts a normal force on the clothes, causing them to follow a curved path by Newton's second law. Where the holes in the drum are however, this normal force cannot act, because there is no surface. The clothes are too big to move through the holes, but the water in the clothes is not and it will therefore move out through the holes and be removed.

Problem 3: Rocket

The Tsiolkovsky rocket equation is $v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$.

a) A rocket in deep space has a mass 1200 tonnes, of which 1000 tonnes is rocket fuel. The rocket emits exhaust gases at a speed (relative to the rocket body) of 3.8km/s. If the rocket crew want to increase its speed by 200m/s, how much fuel must they burn?

Solution: In the Tsiolkovsky rocket equation, the change in velocity Δv is given by $\Delta v = v_f - v_i$. We can write a positive change in mass as $\Delta m = m_i - m_f$ since the mass of rocket before is greater than afterwards. We can then rearrange the Tsiolkovsky rocket equation algebraically to find

$$\Delta m = m_i\left(1 - e^{-\frac{\delta v}{v_e}}\right) = 1200\left(1 - e^{-\frac{200}{3800}}\right) = 61.52 = 62\text{tonnes}, 2\text{sig.fig.} \quad (11)$$

b) The rocket has a payload attached to the front of the rocket. Take the mass of this payload to be equal to your exam candidate number in kilograms. What would be the weight force of this payload when the rocket is standing on the surface of the Earth? (Here take $g = 10\text{ms}^{-2}$.)

Solution: If the exam candidate number is 5999, the answer is 59990N.

c) While cruising at constant speed in deep space, the rocket is able to separate the payload using an explosive charge that provides a total impulse of $I = \int F dt = 0.03$Ns. This explosive charge separates the payload from the rocket in the direction of travel, such that after the separation the new velocity of the payload is v_1 and the new velocity of the rocket is v_2. What is the change in velocity of the centre of mass of the rocket-plus-payload system after this explosive force has acted?

Solution: By conservation of momentum and Newton's third law, the explosion cannot change the velocity of the centre of mass (it is not acting with a force on anything other than the parts of the rocket, which have equal and opposite changes in momentum.) In this sense the rocket plus payload is a closed system and the change in centre of mass velocity is zero.

d) Show that the relative speed that the front end detaches from the back end is given by

$$|v_1 - v_2| = I\left(\frac{1}{m_2} + \frac{1}{m_1}\right). \tag{12}$$

Solution: Both parts of the rocket have the same velocity initially. The explosion will act on both parts of the rocket with the same impulse, giving momenta changes in opposite directions

$$\begin{aligned} I &= m_1 \Delta v_1 \\ I &= -m_2 \Delta v_2 \end{aligned} \tag{13}$$

This gives

$$\Delta v_1 - \Delta v_2 = \frac{I}{m_1} + \frac{I}{m_2} = I\left(\frac{1}{m_2} + \frac{1}{m_1}\right) \tag{14}$$

Putting $\Delta v_1 - \Delta v_2 = |v_1 - v_2|$ we obtain the result, without needing to specify whether v_1 is larger or smaller than v_2 which would be equivalent to specifying whether m_1 is smaller or larger than m_2.

Problem 4: A block and a spring

The formula for a simple harmonic oscillator is $x(t) = A\cos(\omega t + \phi)$

a) A block of mass 4.3kg is dropped from 3.6m above the ground onto an initially unstretched massless spring of spring constant 26kNm^{-1} and equilibrium length 0.40m. How long does it take until the block comes into contact with the spring? Recall that there is gravity of $g = 9.8\text{ms}^{-2}$.

Solution: The block falls under gravity, and we neglect air resistance. So the time taken to fall is obtained from a kinematic relation $d =$

$ut + \frac{1}{2}at^2$ **where** $d = 3.6 - 0.4 = 3.2$**m and we have**

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 * (3.6 - 0.4)}{9.8}} = 0.808 = 0.81\text{s} \tag{15}$$

b) Neglecting any disspative effects, how far does the spring compress from its unstretched position when the block falls on it? Recall that there is gravity of $g = 9.8\text{ms}^{-2}$.

Solution: Use conservation of energy. When the block is dropped all the energy is gravitational potential energy (kinetic energy is zero) and the block is a distance $d = 3.2$**m from the spring. When the spring is maximally compressed all the energy is spring potential energy (kinetic energy is zero and we can take the zero of gravitational potential energy to be the point of maximum compression.) This gives**

$$mg(d + x) = \frac{1}{2}kx^2 \tag{16}$$

This is a quadratic equation for the displacement of the spring x**. We need the positive root since the spring compresses rather than expands.**

$$\begin{aligned} x_{compress} &= \frac{mg + \sqrt{(mg)^2 + 2mgkd}}{k} \\ &= \frac{4.3 * 9.8 + \sqrt{(4.3 * 9.8)^2 + 2 * 4.3 * 9.8 * 26000 * 3.2}}{26000} \\ &= 0.1035\text{m} \end{aligned} \tag{17}$$

So to 2 significant figures, the spring compresses 10**cm.**

c) How long does it take from the moment the block contacts the spring until the spring is maximally compressed?

Solution: The motion as the spring compresses is simple harmonic motion around the equilibrium point. The equilibrium point is where there is no net force on the block and the spring force equals the force of gravity. This occurs where $x = \frac{mg}{k} = \frac{4.3*9.8}{26000} = 0.0016$**m.**

From the equilibrium point until the maximum compression, the spring executes a quarter of a period. The angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{26000}{4.3}} = 77.76\text{s}^{-1}$**. The period is therefore**

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = \frac{2 * 3.14159}{77.76} = 0.081\text{s} \tag{18}$$

So the time taken to reach compression is $t_{compress} = \frac{T}{4} = 0.020\text{s}$ to 2 significant figures.

The time taken from contact with the spring to the equilibrium point $t_{contact}$ can be found from

$$t_{contact} = \sin^{-1}\left(\frac{x_{contact}}{x_{compress} - x_{contact}}\right)\frac{1}{\omega} \tag{19}$$

This is a small number, $t_{contact} = 0.0002\text{s}$, not enough to change the above result at 2 significant figures.

d) Describe in words what would happen after the spring reaches its maximal compression. Does the motion sound realistic? What real-world effects are important to include in order to describe a realistic situation?

Solution: We have assumed conservation of energy with no dissipation. This implies the motion is reversible and after maximum compression the spring will re-expand and launch the block back up into the air to its original height, whereafter the block will fall again and the motion will repeat. This doesn't sound very realistic and in the real world we would expect dissipative effects to drain energy out of the system, such as air resistance and dissipation in the spring from heating. This would lead to the block settling at the gravitational equilibrium point, (40-0.16)cm above the ground.

We have assumed that the centre of mass of the block is exactly aligned with the axis of the spring. If it is not, the block may spring off in a different direction than exactly vertical. We have additionally assumed that the spring is rigidly attached to the ground. It can neither break off from the ground, nor be embedded further into the ground when the block compresses it (the normal force between block and ground can take any value). For precision calculations, we would need to include the variation of gravity and the rotation of the Earth.

FYS100 Physics – Formula sheet

Rotational motion about a fixed axis	Translational motion
Angular velocity $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Translational velocity $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Angular acceleration $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Translational acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Net torque $\sum_k \tau_k = I\,\alpha$	Net force $\sum_k F_k = m\,a$
$\alpha = \text{constant} \begin{cases} \omega_f = \omega_i + \alpha\,t \\ \theta_f = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\,t \end{cases}$	$a = \text{constant} \begin{cases} v_f = v_i + a\,t \\ x_f = x_i + v_i\,t + \frac{1}{2}a\,t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\,t \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Work $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Rotational kinetic energy $K = \frac{1}{2}\,I\,\omega^2$	Kinetic energy $K = \frac{1}{2}\,m\,v^2$
Power $\mathcal{P} = \tau\,\omega$	Power $\mathcal{P} = F\,v$
Angular momentum $L = I\,\omega$	Linear momentum $p = m\,v$
Net torque $\sum_k \tau_k = \frac{\mathrm{d}L}{\mathrm{d}t}$	Net force $\sum_k F_k = \frac{\mathrm{d}p}{\mathrm{d}t}$

General formulas	
Motion with constant acceleration	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\,t + \frac{1}{2}\,\vec{\mathbf{a}}\,t^2 \end{cases}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = m\,\vec{\mathbf{a}}$
Work	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Work-kinetic energy theorem	$\Delta K = W$
Linear momentum	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$
Impulse	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impulse-momentum theorem	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Center of mass	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \int \vec{\mathbf{r}}\,\mathrm{d}m$
Moment of inertia	$I = \int r^2\,\mathrm{d}m$
Parallel-axis theorem	$I = I_{\text{CM}} + M\,D^2$
Torque	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Angular momentum	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Net torque	$\sum_k \vec{\boldsymbol{\tau}}_k = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$
Rotational motion	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonic oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Mathematical rules

Vector relations	
Scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \cos\phi$
Magnitude of vector product	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \sin\phi$

Trigonometry	
Definitions	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identities	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\sin\alpha\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Derivatives	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

Quadratic equations	
Equation	$a\,t^2 + b\,t + c = 0$
Solution	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Equation of a straight line	
Two points on the line are given	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Oppgave 1: Bobslede

a) En bobslede med en vekt på 210kg blir skyvet fra hvile med en konstant akselerasjon av to løpere med en total masse på 180kg. Løperne skyver bobsleden en avstand på 48 meter på en tid på $2,8$ sekunder. Ta koeffisienten for kinetisk friksjon mellom isen og bobsleden til å være $0,02$. Hva er den initiale kraften som utøves på bobsleden av de to løperne når først bobsleden beveger seg?

b) Løperne hopper deretter raskt inn i bobsleden og den beveger seg nedover et rett spor med lengde 52m som skråner 11 grader til horisontalen. Tegn et tydelig diagram over situasjonen, som indikerer retningen til de relevante kreftene på bobsleden. Bruk den tilgjengelige informasjonen til å beregne hastigheten på bobsleden i bunnen av skråningen. Oppgi eventuelle approksimasjoner du gjør for å beregne dette.

c) Etter å ha nådd bunnen av skråningen går bobsleden inn i en sving på et jevnt underlag. Radiusen til denne svingen er 36m. Hvilken vinkel i forhold til vertikalen må bobsleden vippe på (ved å kjøre oppe på et buet spor) for å gjøre en slik bøyning uten ekstra styring?

d) Beskriv med ord hvorfor det kan være en fordel å ha tyngre løpere for å komme raskere nedover skråningen.

Oppgave 2: Vaskemaskin

a) Trommelen til en vaskemaskin har en radius på 25cm. Trommelen roteres fra hvile av en elektrisk motor til den roterer 11 ganger per sekund. Det tar $0,50$ sekunder å spinne opp trommelen. Hvilket konstant dreiemoment må den elektriske motoren bruke hvis vaskemaskinen er fylt med våte klær som veier $4,0$kg, og vi antar at klærne fordeler seg jevnt rundt kanten på vaskemaskinens trommel? Treghetsmomentet til et tynt sylindrisk skall er $I = MR^2$.

b) Forutsatt at den elektriske motoren bruker et konstant dreiemoment, gjennom hvilken vinkel roterer vaskemaskinens trommel i de $0,50$ sekunder det tar å spinne opp trommelen?

c) Hvis vaskemaskinen i stedet er fylt med 8kg våte klær, forklar hva som ville skje annerledes enn med 4kg våte klær og hvorfor. Vil det ta lengre eller kortere tid å spinne opp trommelen? Ville den elektriske motoren bruke mer eller mindre energi? Anta at den elektriske motoren bruker samme dreiemoment, og at trommelen spinnes opp til samme rotasjonshastighet.

d) Under den raske rotasjonen av trommelen fjernes vann fra klærne gjennom små hull i veggene på trommelen. Forklar med hjelp av Newtons bevegelseslover og bevaringslover hvorfor vann kan fjernes på denne måten.

Oppgave 3: Rakett

Tsiolkovskys rakettligning er $v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$.

a) En rakett i det dype verdensrommet har en masse på 1200 tonn, hvorav 1000 tonn er rakettdrivstoff. Raketten avgir eksosgasser med en hastighet (i forhold til raketten) på $3,8$km/s. Hvis rakettmannskapet vil øke hastigheten med 200m/s, hvor mye drivstoff må de forbrenne?

b) Raketten har en nyttelast festet til fronten av raketten. Ta massen av denne nyttelasten til å være lik eksamenskandidatnummeret ditt i kilogram. Hva ville vektkraften til denne nyttelasten være når raketten står på jordoverflaten? (Her tar du $g = 10\text{ms}^{-2}$.)

c) Mens den kjører med konstant hastighet i det dype rommet er raketten i stand til å skille seg bort fra nyttelasten ved hjelp av en eksplosiv ladning som gir en total impuls på $I = \int F dt = 0,03\text{Ns}$. Denne eksplosive ladningen skiller nyttelasten fra raketten i kjøreretningen, slik at den nye hastigheten til nyttelasten etter separasjonen er v_1 og den nye hastigheten til raketten er v_2. Hva er hastighetsendringen av massesenteret til rakett-pluss-nyttelast systemet etter at denne eksplosive kraften har virket?

d) Vis at den relative hastigheten som nyttelasten løsner seg fra raketten med er gitt av

$$|v_1 - v_2| = I\left(\frac{1}{m_2} + \frac{1}{m_1}\right). \tag{20}$$

Oppgave 4: En blokk og en fjær

Formelen for en enkel harmonisk oscillator er $x(t) = A\cos(\omega t + \phi)$

a) En blokk med en masse på $4,3$kg slippes fra $3,6$ meter over bakken til en opprinnelig ustrukket masseløs fjær med fjærkonstant 26kNm^{-1} og likevektslengde

$0,40$m som er festnet til bakken. Hvor lang tid tar det til blokken kommer i kontakt med fjæren? Husk at det er tyngdekraft, $g = 9,8\text{ms}^{-2}$.

b) Ved å neglisjere eventuelle dissipative effekter, hvor langt komprimerer fjæren seg fra sin ustrukket posisjon når blokken faller på den? Husk at det er tyngdekraft, $g = 9,8\text{ms}^{-2}$.

c) Hvor lang tid tar det fra det øyeblikket blokken kommer i kontakt med fjæren til fjæren er maksimalt komprimert?

d) Beskriv med ord hva som ville skje etter at fjæren kom til maksimal kompresjon. Høres bevegelsen realistisk ut? Hvilke virkelige effekter er det viktig å ha med for å beskrive en realistisk situasjon?

FYS100 Fysikk – formelark

Rotasjon om en fast akse	**Éndimensjonal bevegelse**
Vinkelhastighet $\omega = \frac{d\theta}{dt}$	Hastighet $v = \frac{dx}{dt}$
Vinkelakselerasjon $\alpha = \frac{d\omega}{dt}$	Akselerasjon $a = \frac{dv}{dt}$
Resultantmoment $I\,\alpha = \sum_k \tau_k$	Resultantkraft $m\,a = \sum_k F_k$
$\alpha = \text{konstant} \begin{cases} \omega_f = \omega_i + \alpha\, t \\ \theta_f = \theta_i + \omega_i\, t + \frac{1}{2}\alpha\, t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\, t \end{cases}$	$a = \text{konstant} \begin{cases} v_f = v_i + a\, t \\ x_f = x_i + v_i\, t + \frac{1}{2} a\, t^2 \\ v_f^2 = v_i^2 + 2\, a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\, t \end{cases}$
Arbeid $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Arbeid $W = \int_{x_i}^{x_f} F\, dx$
Kinetisk energi $K = \frac{1}{2}\, I\, \omega^2$	Kinetisk energi $K = \frac{1}{2}\, m\, v^2$
Effekt $\mathcal{P} = \tau\, \omega$	Effekt $\mathcal{P} = F\, v$
Spinn $L = I\, \omega$	Bevegelsesmengde $p = m\, v$
Spinnsatsen $\frac{dL}{dt} = \sum_k \tau_k$	Newtons 2. lov $\frac{dp}{dt} = \sum_k F_k$

Generelle sammenhenger	
Bevegelse med konstant akselerasjon	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\, t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\, t + \frac{1}{2}\, \vec{\mathbf{a}}\, t^2 \end{cases}$
Newtons 2. lov	$m\, \vec{\mathbf{a}} = \sum_k \vec{\mathbf{F}}_k$
Arbeid	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$
Arbeid-kinetisk energi teoremet	$\Delta K = W$
Bevegelsesmengde	$\vec{\mathbf{p}} = m\, \vec{\mathbf{v}}$
Newtons 2. lov	$\frac{d\vec{\mathbf{p}}}{dt} = \sum_k \vec{\mathbf{F}}_k$
Impuls	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\, dt$
Impuls-bevegelsesmengde teoremet	$\Delta \vec{\mathbf{p}} = \vec{\mathbf{I}}$
Massesenter	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \int \vec{\mathbf{r}}\, dm$
Treghetsmoment	$I = \int r^2\, dm$
Steiners sats (parallellakseteoremet)	$I = I_{\text{CM}} + M\, D^2$
Kraftmoment	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Spinn	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Spinnsatsen	$\frac{d\vec{\mathbf{L}}}{dt} = \sum_k \vec{\boldsymbol{\tau}}_k$
Sirkelbevegelse	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonisk oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Matematiske sammenhenger

Vektorrelasjoner	
Prikkprodukt	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \|\vec{\mathbf{A}}\|\,\|\vec{\mathbf{B}}\| \cos\phi$
Absoluttverdi av kryssprodukt	$\|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\| = \|\vec{\mathbf{A}}\|\,\|\vec{\mathbf{B}}\| \sin\phi$

Trigonometri	
Definisjoner	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identiteter	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Deriverte	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

2. grads ligning	
Ligning	$a\,t^2 + b\,t + c = 0$
Løsning	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Ligningen for en rett linje	
Gitt to punkter på linjen	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$