

$$1) a) H(j\omega) = \frac{2(-3j\omega + 4)}{(j\omega + 1)(3j\omega + 3)}$$

$$= \frac{8 - 6j\omega}{(1 + j\omega)(3 + j\omega)}$$

$$|H(j\omega)| = \frac{\sqrt{8^2 + 6^2\omega^2}}{\sqrt{1 + \omega^2} \cdot \sqrt{3^2 + \omega^2}}$$

$$\angle H(j\omega) = \text{atan}\left(-\frac{6\omega}{8}\right) - \text{atan}(\omega) - \text{atan}(\omega)$$

b) Polene er begge i  $p_{1,2} = -1$

$$(s+1)(3s+3) = (s+1)(s+1) \cdot 3 = 0$$

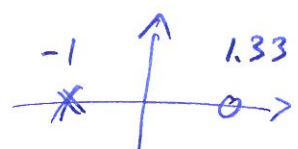
$$s_1, s_2 = -1$$

Asymptotisk stabilt

Kritisk dempet, sammenfallende poler

Nullplass i  $-3s + 4 = 0$

$$s = \frac{4}{3}$$



c)  $\xrightarrow{\omega}$

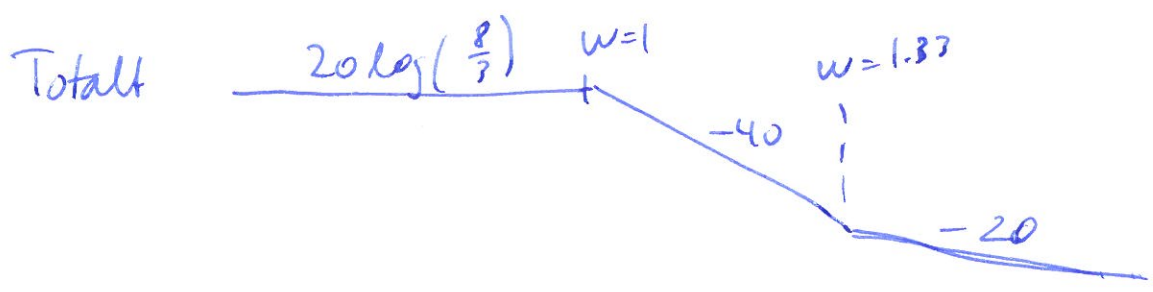
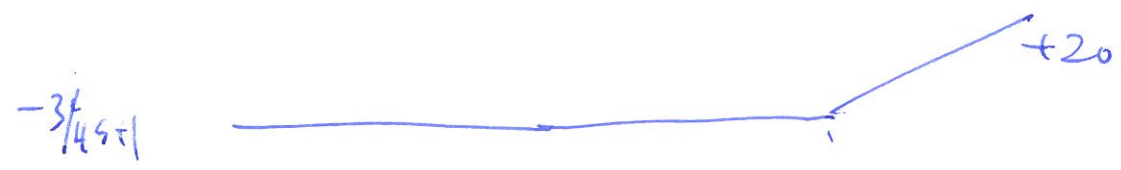
Finer fort  $H(s)$  pi standard form.

$$H(s) = \frac{4 \cdot 2 \left(-\frac{3}{4}s + 1\right)}{(s+1)(s+1) \cdot 3}$$

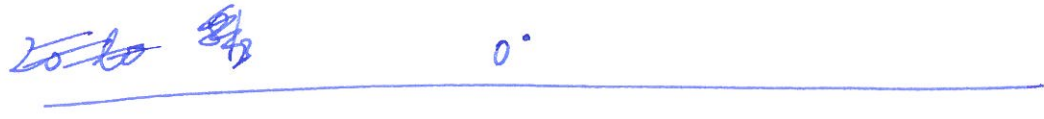
$$= \frac{\frac{8}{3} \left(-\frac{3}{4}s + 1\right)}{(s+1)(s+1)}$$

$$\omega_k = 1 \quad \omega_k = \frac{1}{3/4} = 4/3$$

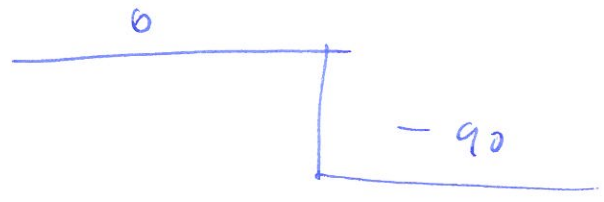
$20 \log\left(\frac{8}{3}\right)$



(f) (pts)



$\angle \frac{8}{3}$



$\angle \frac{1}{s+1}$



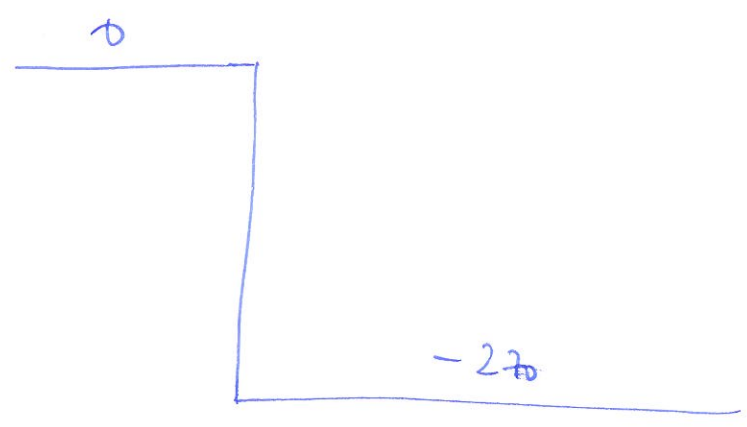
$\angle \frac{1}{s+1}$



$\angle \frac{-3}{4s+1}$



Total



d)

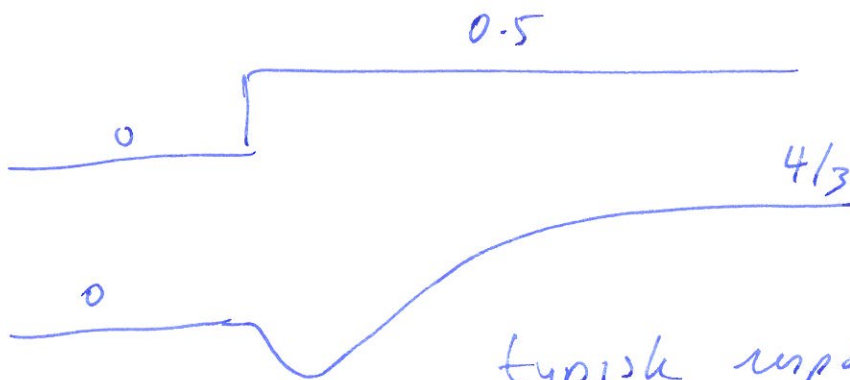
$$u(t) = 0.5$$

Benyttes slutte verdi teorem:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot y(s) = \lim_{s \rightarrow 0} (s \cdot H(s) \cdot \frac{0.5}{s})$$

$$= H(0) \cdot 0.5$$

$$= \frac{8}{3} \cdot 0.5 = \underline{\underline{\frac{4}{3}}}$$



typisk respons for  
system med nullpnt  
i h.h.p.

Oppg 2)

a) Bare negative realdele ( $\Rightarrow$  asymptotisk stabil)

$$b) \dot{x}_1 = 2(u - 4x_1 - x_2)$$

$$\dot{x}_1 = -8x_1 - 2x_2 + 2u$$

$$\dot{x}_2 = \frac{1}{2}(2x_1 - 2x_2)$$

$$\dot{x}_2 = x_1 - x_2$$

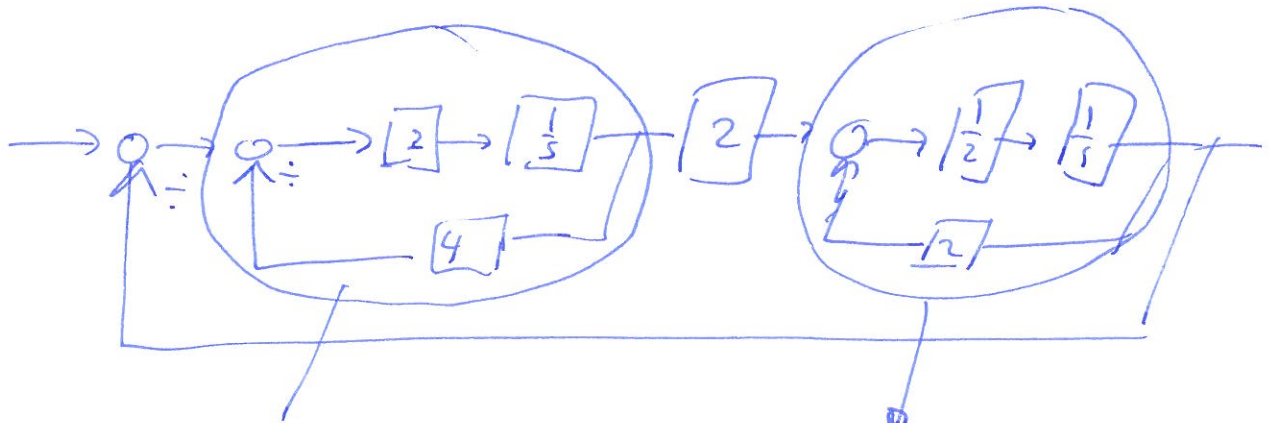
$$y = x_2$$

$$c) A = \begin{bmatrix} -8 & -2 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

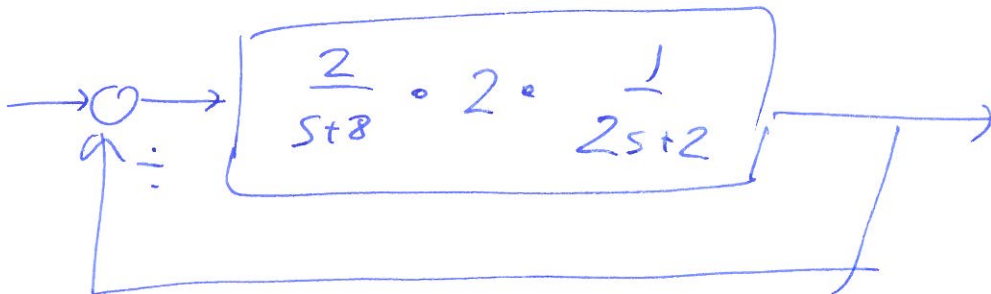
$$D = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

d) ~~Ketten~~ Blockdiagramm manipulation ⑥



$$\frac{\frac{2}{s}}{1 + \frac{2}{s} \cdot 4} = \frac{2}{s+8}$$

$$\frac{\frac{1}{2s}}{1 + \frac{1}{2s} \cdot 2} = \frac{1}{2s+2}$$



$$H(s) = \frac{4}{(s+8)(2s+2)}$$

$$1 + \frac{4}{(s+8)(2s+2)} \cdot 1$$

$$= \frac{4}{(s+8)(2s+2) + 4} = \frac{4}{2s^2 + 18s + 20}$$

(7)

e) Pole

$$2s^2 + 18s + 20 = 0$$

$$s_{1,2} = \frac{-18 \pm \sqrt{18^2 - 4 \cdot 2 \cdot 20}}{2 \cdot 2}$$

$$= \frac{-18 \pm \sqrt{324 - 160}}{4}$$

$$= \frac{-18 \pm 12.8}{4}$$

$$\underline{s_1 = -7.7} \quad \underline{s_2 = -1.3}$$

Bege er i v.h-p  $\Rightarrow$  asympt. stab.

Stemmer med 2a)

Overdamped system,  $\zeta > 1$ , pga

to reelle poler. Kan vises.

0 Pgs 3

Ⓟ

$$a) \frac{dE(t)}{dt} = \sum Q_{\text{in}}(t) - \sum Q_{\text{out}}(t)$$

$$b) E(t) = m \cdot c_p \cdot T(t)$$

$$Q_{\text{in}}(t) = P(t)$$

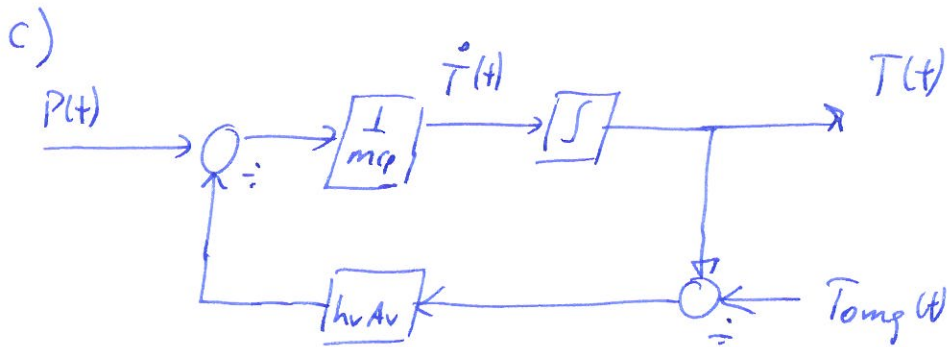
$$Q_{\text{out}}(t) = h_r \cdot A_r (T(t) - T_{\text{amb}}(t))$$

$$m \cdot c_p \cdot \frac{dT(t)}{dt} = P(t) - h_r A_r (T(t) - T_{\text{amb}}(t))$$

$$\frac{dT(t)}{dt} = \frac{1}{m \cdot c_p} (P(t) - h_r A_r (T(t) - T_{\text{amb}}(t)))$$

antar konstant  $m$ ,  $c_p$ ,  $h_r \cdot A_r$





d)  $s \cdot \overline{T}(s) = \frac{1}{mc} \left( P(s) - hvAv (\overline{T}(s) - \overline{T}_{omg}(s)) \right)$

$$\left( s + \frac{hvAv}{mc} \right) \overline{T}(s) = \frac{1}{mc} P(s) + \frac{hvAv}{mc} \overline{T}_{omg}(s)$$

Finnes  $H_p(s)$  när  $\overline{T}_{omg}(s) = 0$

$$\begin{aligned} H_p(s) &= \frac{\overline{T}(s)}{P(s)} = \frac{1/mc}{s + \frac{hvAv}{mc}} \\ &= \frac{1}{mc \cdot s + hvAv} \\ &= \frac{1/hvAv}{\frac{mc}{hvAv} s + 1} \end{aligned}$$

(10)

e)  $H_v(s)$  finner vi når  $P(s) = 0$

$$H_v(s) = \frac{\overline{T_{omg}(t)}}{\overline{P}} \frac{T(s)}{\overline{T_{omg}(s)}}$$

$$= \frac{\frac{h\nu A_v}{mC}}{s + \frac{h\nu A_v}{mC}} = \frac{1}{\frac{mC}{h\nu A_v} s + 1}$$

f) • ↑ oven begge to

•  $K = 1/h\nu A_v$ ,  $T = \frac{mC}{h\nu A_v}$

•  $K_v = 1$ ,  $T_v = \frac{mC}{h\nu A_v}$

↑  
sammenheng mellom temperatur <sup>i forstymelse</sup>  $\overline{T_{omg}(t)}$   
og utgang  $\overline{T}(t)$ . Samme beregning.

$$\overline{T_{omg}} = 10^\circ\text{C}, P(t) = 0 \Rightarrow \overline{T}(t) = 10^\circ\text{C}$$

$$K_v = 1$$

(11)

$$g) H_0(s) = H_r(s) \cdot H_p(s) \cdot H_m(s)$$

$$= K_p \cdot \frac{1/hvAv}{\frac{mc}{hvAv} s + 1} \cdot \frac{1}{\underbrace{30s + 1}_{H_m(s)}}$$

$$T_m = 30 \text{ sek}$$

$$\omega_b = \frac{1}{30} = 0.033$$

$$M(s) = \frac{H_0(s)}{1 + H_0(s)} = \frac{\frac{t}{n}}{1 + \frac{t}{n}} = \frac{t}{n + t}$$

$$= \frac{K_p \cdot \frac{1}{hvAv} \cdot 1}{\left(\frac{mc}{hvAv} s + 1\right)(30s + 1) + K_p \cdot \frac{1}{hvAv} \cdot 1}$$

$$N(s) = \frac{1}{1 + H_0(s)} = \frac{1}{1 + \frac{t}{n}} = \frac{n}{n + t}$$

$$= \frac{\left(\frac{mc}{hvAv} s + 1\right)(30s + 1)}{\left(\frac{mc}{hvAv} s + 1\right)(30s + 1) + \frac{1}{hvAv} \cdot K_p \cdot 1}$$

$$\left(\frac{mc}{hvAv} s + 1\right)(30s + 1) + \frac{1}{hvAv} \cdot K_p \cdot 1$$

h) Følgesystemet er ikke så  
gode med P-reg.

Regulatoren bliver ikke i følge  
setpunktet. Dette fordi det ikke er en integrator  
i sløjfen.

Reguleringsavviket blir:

$$e(s) = N(s) \cdot y_{ref}(s)$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \cdot e(s)$$

$$= \lim_{s \rightarrow 0} s \cdot N(s) \cdot \frac{T_{ref}}{s}$$

$$= N(0) \cdot T_{ref}$$

$$= \frac{1}{1 + \frac{K_p}{h_v A_v}} \cdot T_{ref}$$

$$= \frac{h_v A_v}{h_v A_v + K_p} \cdot T_{ref}$$


---

$$e(s) = N(s) \cdot y_r(s)$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s e(s)$$

$$= \lim_{s \rightarrow 0} s N(s) \cdot y_r(s)$$

$$= \lim_{s \rightarrow 0} s \cdot N(s) \cdot \frac{1}{s}$$

$$= \lim_{s \rightarrow 0} \frac{1}{1 + H_0(s)}$$

$$= \frac{1}{1 + H_0(0)}$$

hvor  $H_0(0)$  er  $\approx 4 \text{ dB}$  som er

$$X = 10^{\frac{4}{20}} = 0.63$$

$$\lim_{t \rightarrow \infty} e(t) = \frac{1}{1 + 0.63} = \underline{\underline{0.61}}$$

Fra opg. h)  
forventes:

$$e(t) = \frac{K_v A_v - \bar{r}_n}{K_v A_v + K_p}$$

$$= \frac{0.004}{0.004 + 0.0025} \cdot 1$$

$$= \underline{\underline{0.615}}$$

stemmer  
bra

j) ① Kan la  $K_p \rightarrow \infty$ , men

dette er bare teoretisk interessant. (14)  
② Benytt PI reg.

Eller, ved å la  $h_v A_v \rightarrow 0$  (mer og mer isolasjon)

blir prosessen mer og mer en

integrator, og  $e(t) \rightarrow 0$   
 $t \rightarrow \infty$

Får da en integrator i sløyfa.

k)  $K_p = 0.0025$

$K_p$  må økes med ca **11 dB** ~~11 dB~~  
for å få  $\varphi = 90^\circ$

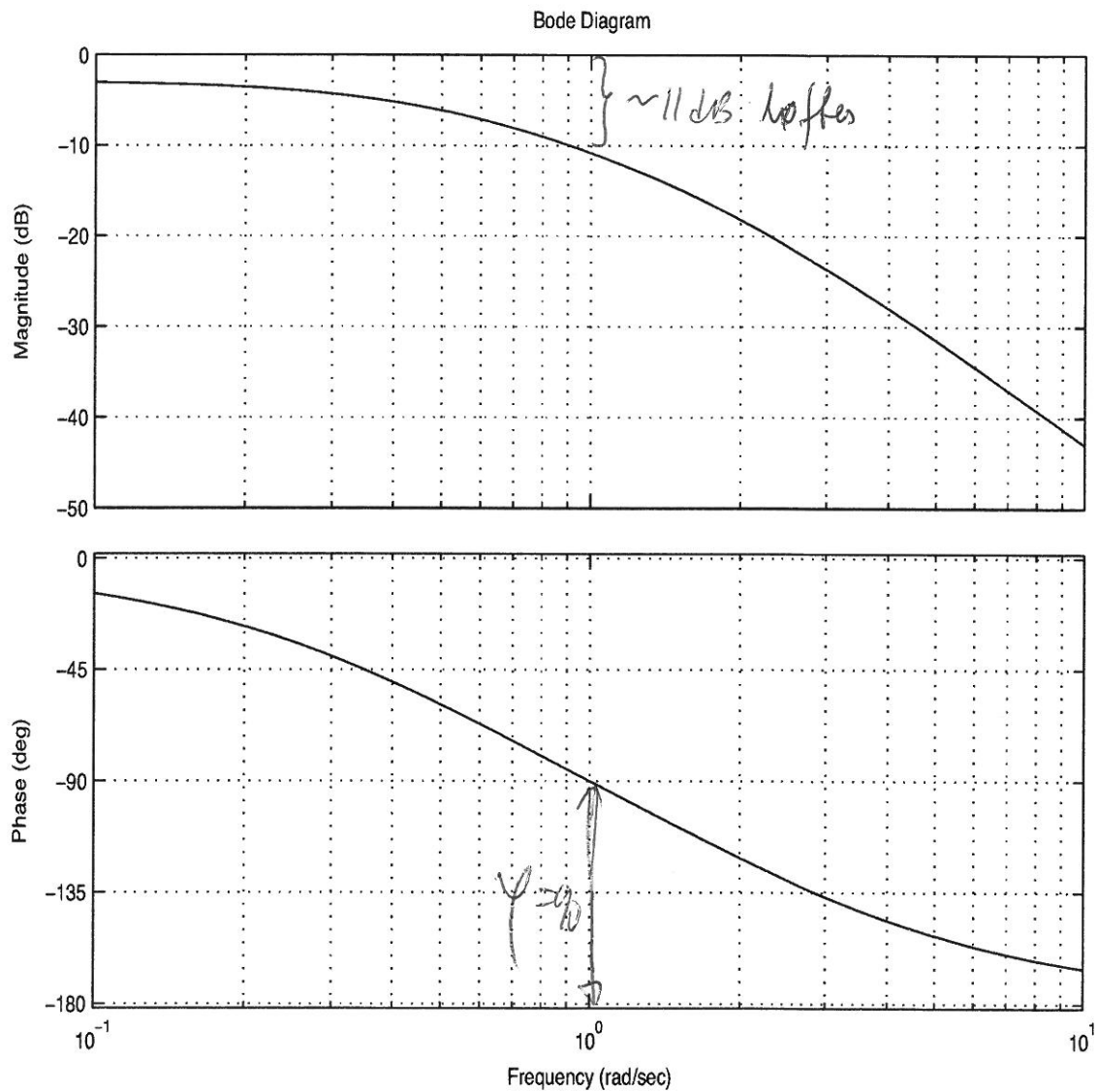
$$\underline{K_{p,ny}} = 0.0025 \cdot 10^{\frac{11}{20}} = \underline{\underline{0.0088}}$$

Vendelig  $\Delta K$ ,  $\underline{\underline{\Delta K = \infty}}$

$$l) H_p(s) = \frac{Y_{h_v A_v}}{m c} = \frac{250}{2500s + 1}$$

$$\underline{\underline{T_r = 1000 \text{ sek}}}$$

Fag: BIE240, Reguleringsteknikk  
Dato: 19. februar 2014  
Kandidatnr:  
Sidenr:



Figur 5: Bodeplot av sløyfetransferfunksjonen  $H_0(j\omega)$  i oppgave 3h).

Velger pol/multiplet løsning

Ti = 2500

$K_p = \frac{2500}{1000 \cdot 250} = \frac{25}{250} = \underline{\underline{0.1}}$

4)

- 1 - C      imaginære poler  
marginalt stab
- 2 - E      reelle poler, overdempet
- 3 - A      multiplet i V.H.P.  
oversving, starter som  
1 orden
- 4 - B      instabilt, 2 integratorer
- 5 - D      multiplet i H.H.P.  
invers respons.