Løsningsforslag 9/12 - 2010 Reg.tek

a) $2s^2+3s+4=0$

$$s=\frac{-3\pm\sqrt{9-4\cdot 4\cdot 2}}{2\cdot 2}$$

$$=\frac{-3\pm\sqrt{-7}}{4}$$

$$=-\frac{3}{4}\pm j\frac{\sqrt{7}}{4}$$

Systemet er asymptotisk stabilt

$$2s^2+3s+4 = \frac{1}{2}s^2+\frac{3}{4}s+1 = \frac{s^2}{w_0^2}+\frac{2\zeta}{w_0}s+1$$

$$w_0=\sqrt{2}$$

$$\frac{2\zeta}{w_0}=\frac{3}{4} \Rightarrow \zeta=\frac{3}{8}\cdot w_0 = \frac{3\sqrt{2}}{8} = 0.53$$

$\zeta<1 \Rightarrow$ underdamped

b) $$H(s) = \frac{1}{2(j\omega)^2 + 3j\omega + 4}$$

$$= \frac{1}{(4-2\omega^2) + j3\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(4-2\omega^2)^2 + (3\omega)^2}}$$

$$\angle H(j\omega) = \cancel{\text{atan}}\ \text{atan}\left(\frac{3\omega}{(4-2\omega^2)}\right)$$

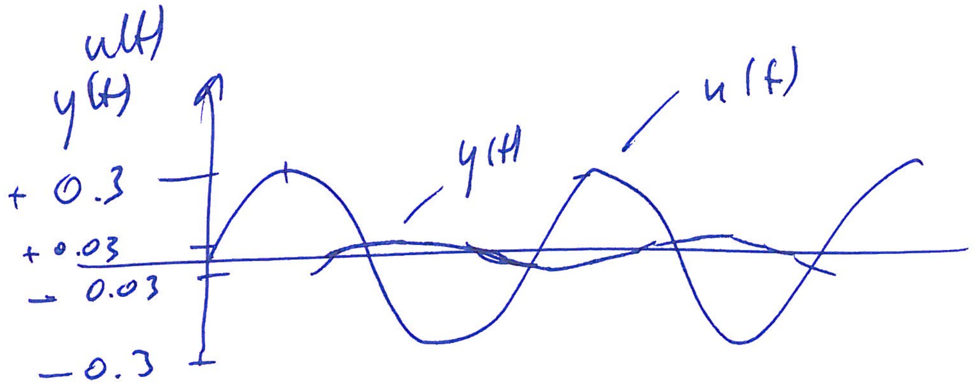
c) ved $\omega = 2.4$ er $|H(j\omega)| = -20\,dB$

$$\angle H(j\omega) = -135°$$

$\underline{-20\,dB = 0.1}$

$$y(t) = 0.1 \cdot 0.3 \cdot \sin(2.4t - 135°)$$

$$= 0.03 \sin(2.4t - 135°)$$



d) $K = \frac{1}{4}$

Denne finnes i fig 1 når $\omega \to 0$

$20 \log\left(\frac{1}{4}\right) = -12\ dB$

Avlest ω_b er ved $|H(j\omega)| = -15\ dB$

~~[illegible]~~ $\omega_b \approx 1.8\ rad/sek$

~~e) $\zeta = 0.53 \Rightarrow \delta = 0.13$ (ca)~~

~~$\Rightarrow$ 13% oversving, 13% mer enn~~

~~$y_s = K \cdot u_s$~~

~~$\Downarrow$~~

~~$y_{max} = y_s \cdot (1 + 0.13)$~~ ~~$\neq$~~ ~~$y_s = 0.25 \cdot 1 = 0.25$~~

~~0.2825~~

(4)

e) Responstid:

$$T_r \approx \frac{1.5}{\omega_0} \quad \text{eller} \quad T_r \approx \frac{1.5}{2\omega_b} \quad \text{eller}$$

$$T_r \approx \frac{1}{\omega_0}$$

Dette gir:

$$\underline{T_r} \approx \frac{1.5}{\sqrt{2}} = \underline{\underline{1.06 \text{ sek}}}$$

Stasj-verdi i $y(t)$ ved enhetssprang i $u(t)$

$$y_{stasj} = K \cdot u_{stasj}$$

$$= 0.25 \cdot 1 = 0.25$$

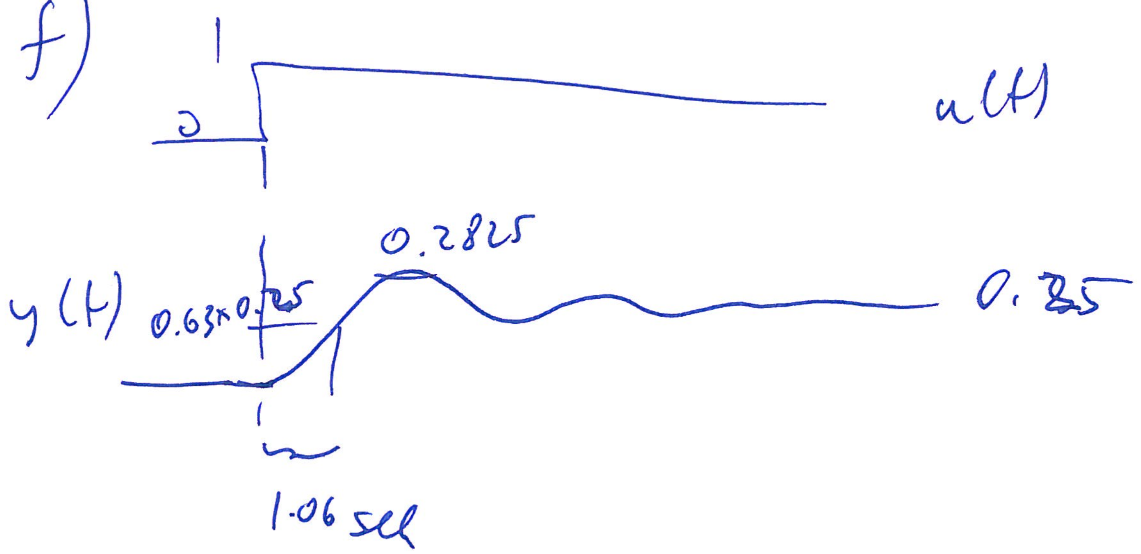
$$\zeta = 0.53 \Rightarrow \delta = 0.13 \quad (\text{ca})$$

$\Rightarrow$ 13% oversving $\Rightarrow$ $\underline{y_{max}} = y_s \cdot (1 + 0.13)$

$$= 0.25 (1.13)$$

$$= \underline{\underline{0.2825}}$$

f)



2 Regulering

a) $K_m = 2$

$H_m(s) = ?$

pol. i $s = -2$ og $s = -10$

2 reelle poler:

$$H_m(s) = \frac{2}{(T_1 s+1)(T_2 s+1)}$$

hvor $T_1 = -\frac{1}{p_1} = -\frac{1}{-2} = \underline{\underline{\frac{1}{2}}}$, $T_2 = -\frac{1}{p_2} = -\frac{1}{-10} = \underline{\underline{0.1}}$

Dette gir

$$H_m(s) = \frac{2}{(0.5s+1)(0.1s+1)}$$

b) $u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(\tau)\,d\tau$

Laplace gir

$$u(s) = K_p e(s) + \frac{K_p}{T_i}\frac{1}{s}e(s)$$

$$\frac{u(s)}{e(s)} = H_r(s) = \dots \frac{K_p + K_p T_i s}{T_i s}$$

$$= \frac{K_p(T_i s + 1)}{T_i s}$$

c) $H_0(s) = H_r(s) \cdot H_p(s) \cdot H_m(s)$

$$= \frac{K_p(T_i s+1)}{T_i s} \cdot \frac{1}{2s^2+3s+4} \cdot \frac{2}{(0.5s+1)(0.1s+1)}$$

Har ett nullpkt og fjerde orden i nevner, totalt vil $|H(j\omega)|$ falle med $+20-80 = -60$ dB/dek ved høye frekvenser.

Fasen vil være $+90 - 4 \cdot 90 = -3 \cdot 90$
$= \underline{\underline{-270°}}$

ved høye frekvenser.

d) Har $N(s) = \frac{e(s)}{y_r(s)} = \frac{1}{1+H_0(s)}$ ⑧

hvor $e(s) = \frac{1}{1+H_0(s)} \cdot y_r(s)$

hvor $y_r(s) = \frac{1}{s}$

Sluttverditeorem gir

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot e(s)$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{1}{1+H_0(s)} \cdot \frac{1}{\cancel{s}}$$

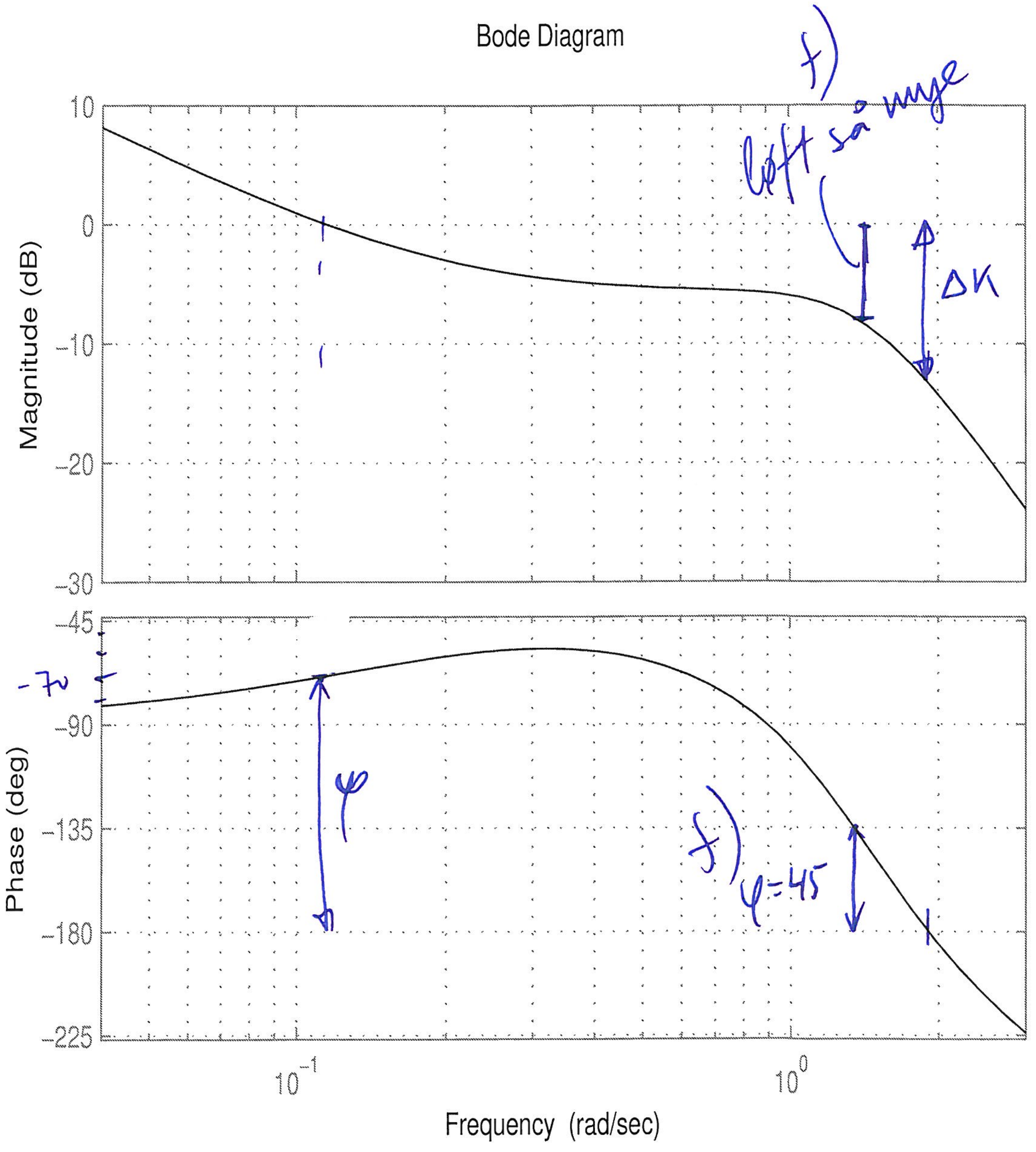
$$= \lim_{s\to 0} \frac{1}{1+H_0(s)}$$

$$H_0(0) = \frac{t_0(0)}{n_0(0)}$$

$$= \frac{1}{1+H_0(0)} = \frac{n_0(0)}{n_0(0)+t_0(0)}$$

$$= \frac{0}{0+2K_p} = \underline{\underline{0}}$$

Fag: BIE240, Reguleringsteknikk
Dato: 9. desember 2010
Kandidatnr:
Sidenr:



Figur 7: Bodeplot av sløyfetransferfunksjonen $H_0(j\omega)$ i oppgave e).

e) Avlest $\underline{\Delta K = 13\ dB}$ (10)

$$\underline{\varphi} = 180 - 70 = \underline{\underline{120^\circ}}$$

f) 25% oversving $\Rightarrow \varphi = 45$

Vil ha fasemargin på 45°

Ser at $\varphi = 45$ avleses ved $\underline{\underline{w\ \cancel{\approx}\ 1.5}}$

Her skal altså w_c $\left(|H_0(jw_c)| = 0\ dB\right)$

være.

Må løfte $|H_0(jw)|$ ca 8 dB

eller ca 2.5 ganger

g) Ny $K_p = 2.5 \cdot$ Gammel K_p

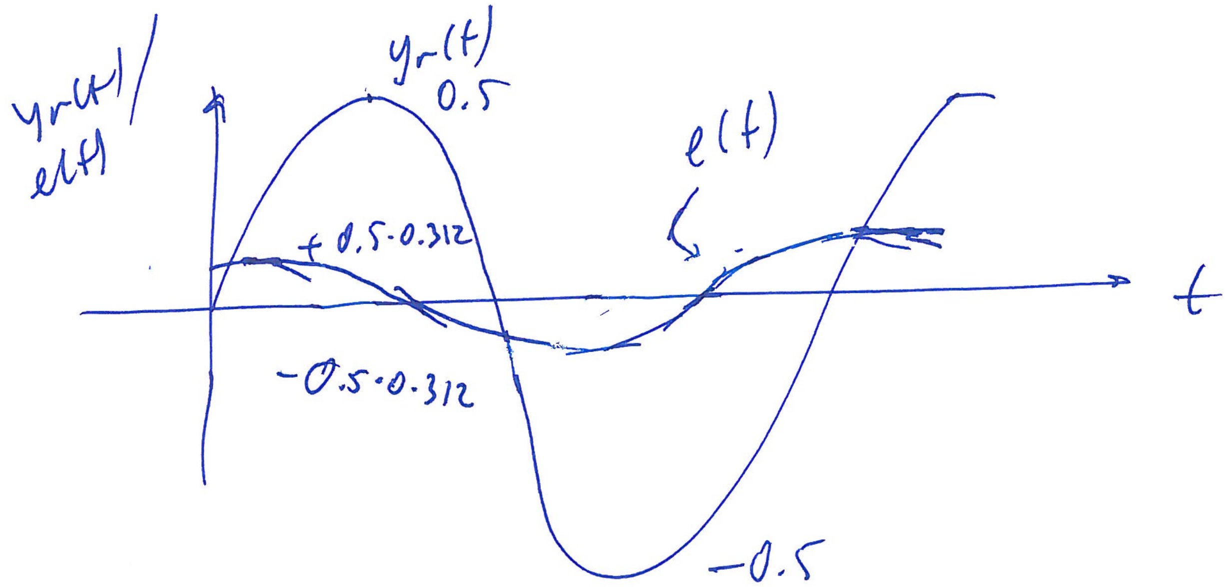
$= 2.5 \cdot 1 = \underline{\underline{2.5}}$

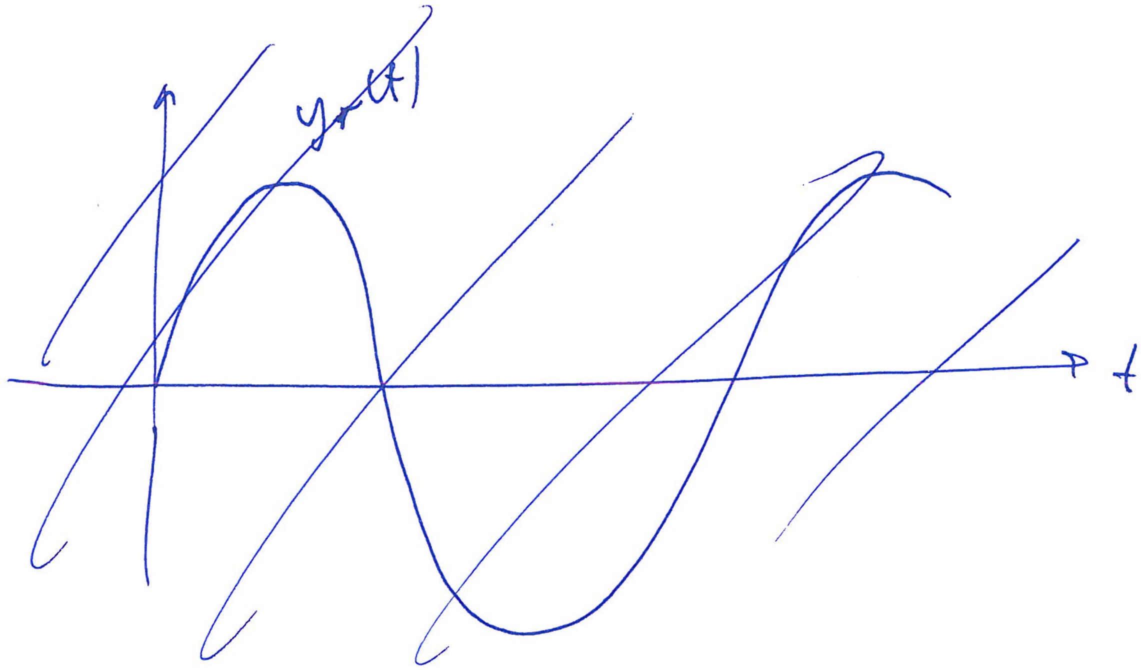
g) Har gitt at $y_r(t) = 0.5 \sin(0.035t)$ (11)

$|N(j\omega)| = -10 \text{ dB} \simeq 0.312$

$e(t) = 0.5 \cdot 0.312 \cdot \sin(0.035t + 70^{\circ})$

fasen ligger faktisk foran...

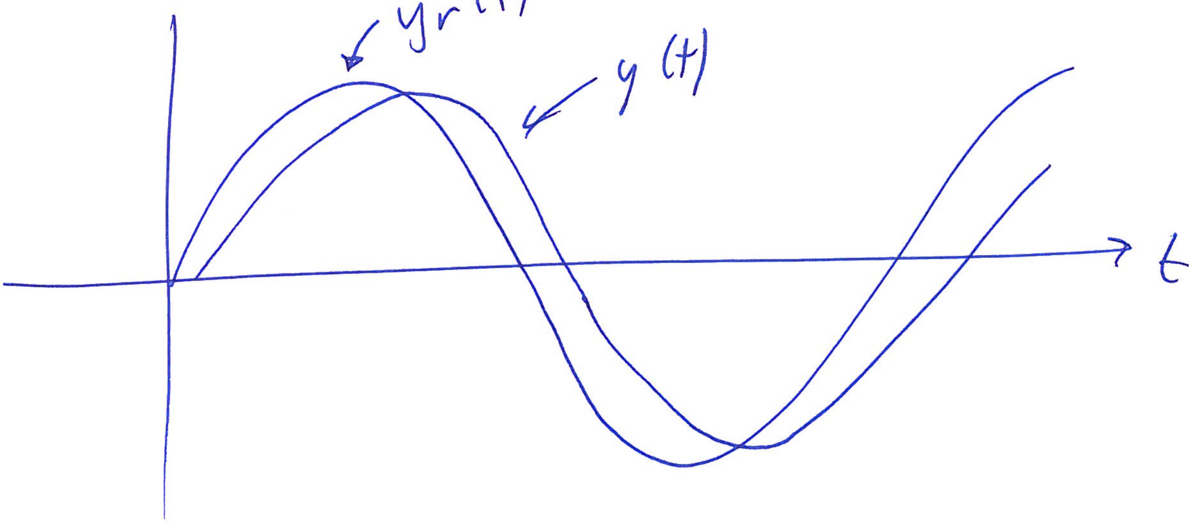




$$\left|M(j0.035)\right| \approx -1\ \text{dB}$$

$$\angle M(j0.035) = -20^\circ$$

} Dette gir



h) N(s) kan tolkes som forholdet mellom

$$\frac{e_{\text{med reg}}(s)}{e_{\text{uten reg}}(s)} = 0.31$$

Regulator er $\frac{1}{0.31} \approx 3.2$ ganger bedre å undertrykke $v(t)$