10 des. 2008

Reg.tek
BIE 240

Løsningsforslag

Oppg 1

a)

$$s = j\omega$$

$$H(j\omega) = \frac{3(j\omega+3)}{(3j\omega+1)(3j\omega+1)}$$

$$= \frac{3(3+j\omega)}{(1+j\omega 3)(1+j\omega 3)}$$

$$= \frac{3\sqrt{3^2+\omega^2}}{\sqrt{1+3^2\omega^2}\sqrt{1+3^2\omega^2}} \cdot e^{j\left(\mathrm{atan}\left(\frac{\omega}{3}\right) - \mathrm{atan}(3\omega) - \mathrm{atan}(3\omega)\right)}$$

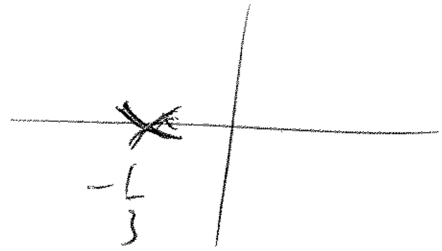
$$|H(j\omega)| = \frac{3\cdot\sqrt{3^2+\omega^2}}{\sqrt{1+9\omega^2}\sqrt{1+9\omega^2}}$$

$$\angle H(j\omega) = \mathrm{atan}\left(\frac{\omega}{3}\right) \div 2\,\mathrm{atan}(3\cdot\omega)$$

2)

b) polene er overlappende i

$$p_1 = p_2 = -\frac{1}{3}$$

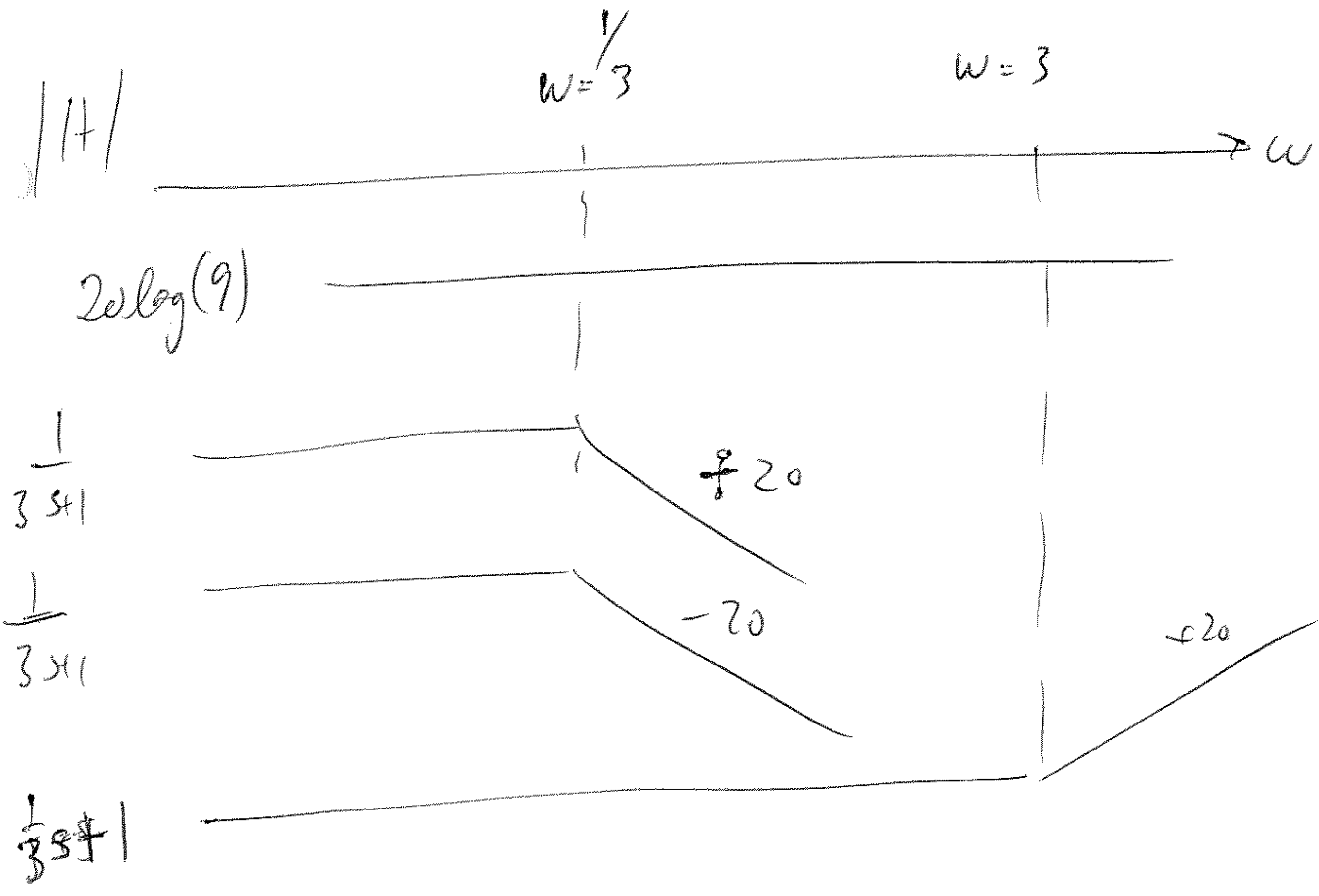


asymptotisk stabilt

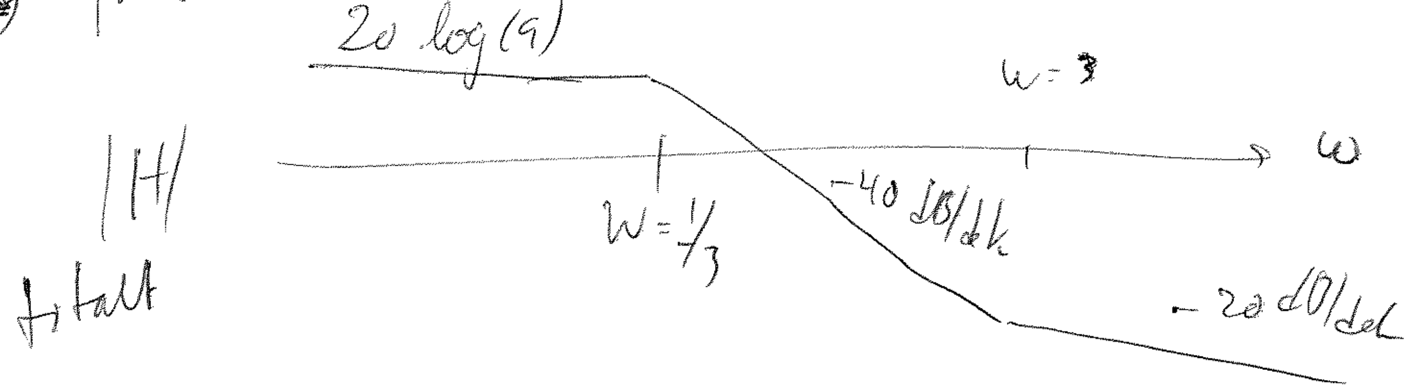
Kritisk dempet

c) Må først skrive om $H(s)$

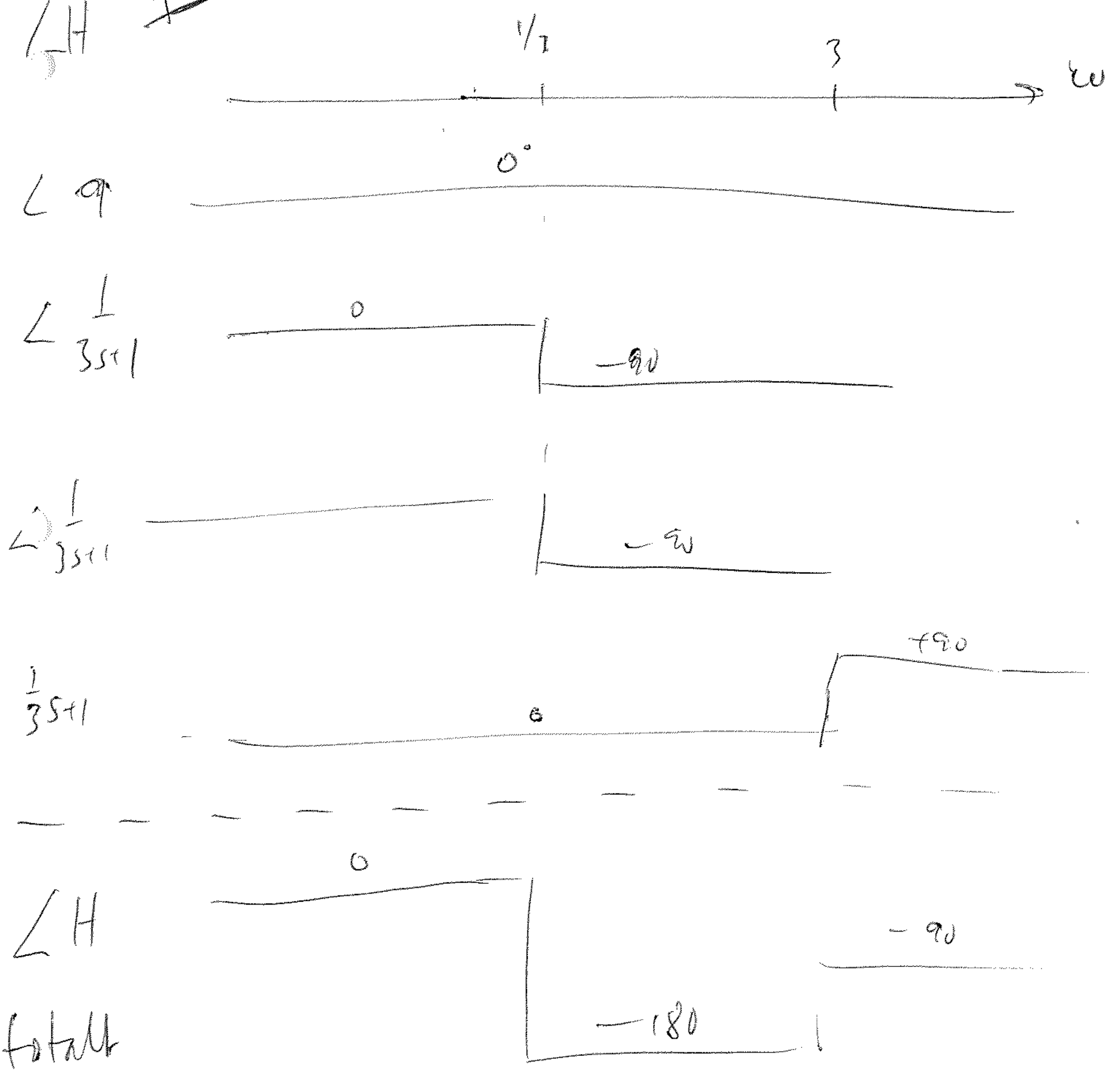
$$H(s) = \frac{9\left(\frac{1}{3}s+1\right)}{(3s+1)(3s+1)}$$



b) forts.


20 log(9)
|H|
totalt
w = 3
ω
W = 1/3
-40 dB/dek
-20 dB/dek

fase


∠H
1/3
3
ω
∠9
0°
∠ 1/(3s+1)
0
-90
∠ 1/(3s+1)
-90
1/(3s+1)
+90
0
∠H
totalt
0
-90
-180

d) $u(t) = 0.5$

Benyttes sluttverditeoremet

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} \left(y(s) \cdot s \right)$$

$$= \frac{3\,(s+3)}{(3s+1)(3s+1)} \cdot \cancel{s} \cdot \underbrace{\frac{0.5}{\cancel{s}}}_{u(s)}$$

$$\lim_{t \to \infty} y(t) = \underline{\underline{4.5}}$$

Oppg 2

a) siden det er positiv tilbakekopling, er det ustabilt

b)
$$\dot{x}_1 = 3(3x_1 - x_2 + u)$$
$$\dot{x}_2 = \frac{1}{2} \cdot (2 \cdot x_1 - 2x_2)$$

$$y = x_2$$

c)
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 9 & -3 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

d)
$$s \cdot x_1(s) = 9x_1(s) - 3x_2(s) + 3u(s) \qquad (\mathrm{I})$$
$$s \cdot x_2(s) = x_1(s) - x_2(s) \qquad (\mathrm{II})$$
$$y(s) = x_2(s) \qquad (\mathrm{III})$$

setter $x_1(s)$ fra (I) inn i (II) og
deretter inn i (III)

$$X_1(s) = \frac{-3}{s-9} X_2(s) + \frac{3}{s-9} u(s) \qquad (I)$$

$$(s+1) \quad X_2(s) = X_1(s) \qquad (II)$$

(I) inn i (II) gir

$$(s+1) \quad X_2(s) = \frac{-3}{(s-9)} X_2(s) + \frac{3}{s-9} u(s)$$

$$(s+1)(s-9) X_2(s) + 3 X_2(s) = 3 u(s)$$

$$\left((s+1)(s-9) + 3\right) X_2(s) = 3 u(s)$$

$$X_2(s) = \frac{3}{s^2 - 8s - 9 + 3} u(s)$$

$$y(s) = \frac{3}{s^2 - 8s - 6} u(s)$$

$$H(s) = \frac{3}{s^2 - 8s - 6}$$

Oppg 3

a) $$\frac{dE}{dt} = E_{ovn}(t) - E_{varmetap}(t)$$

b) antar masse og varmekapasitet konstant

$$E(t) = m\cdot c\cdot T(t)$$

$$\dot{E}_{ovn}(t) = P(t)$$

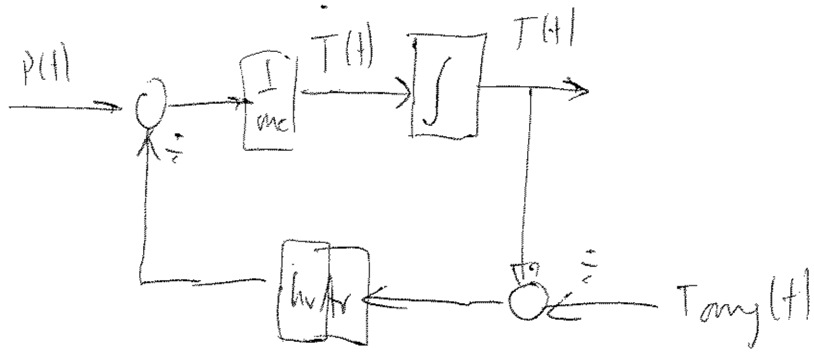
$$E_{varmetap}(t) = h_v A_v \left(T(t) - T_{omg}(t)\right)$$

antar ideell omrøring i rommet
(jevn fordelt temp)

$$\frac{m\cdot c\cdot T(t)}{dt} = P(t) - h_v A_v \left(T(t) - T_{omg}(t)\right)$$

$$\frac{dT(t)}{dt} = \frac{1}{m\cdot c}\left(P(t) - h_v A_v \left(T(t) - T_{omg}(t)\right)\right)$$

c)



d) Laplace gir

$$s \cdot T(s) = \frac{1}{mc} \cdot \left(P(s) - h_v A_v T(s) + h_v A_v T_{omg}(s)\right)$$

$$(m \cdot c s + h_v A_v)\, T(s) = P(s) + h_v A_v T_{omg}(s)$$

$$H_p(s) = \frac{T(s)}{P(s)} = \frac{1}{mcs + h_v A_v} = \frac{1/h_v A_v}{\frac{mc}{h_v A_v} s + 1}$$

e) $$H_v(s) = \frac{T(s)}{T_{omg}(s)} = \frac{h_v A_v}{mcs + h_v A_v} = \frac{1}{\frac{mc}{h_v A_v} s + 1}$$

f) 1orden begge to.

$H_p(s)$: $K = \frac{1}{h_v A_v}$ $\quad T = \frac{mc}{h_v A_v}$

$H_v(s)$: $K = 1$ $\quad T = \frac{mc}{h_v A_v}$

Siden $H_v(s)$ er transferfunksjon
fra temperatur til temperatur er det logisk
at forsterkningen er 1. Øker utetemp med 1 grad
øker innetemp med 1 grad

g) Hvis det plasseres mye møbler i rommet vil m øke.

Dette gjør at tidskonstanten øker.

Dette er logisk fordi det vil ta lenger tid å varme opp både luft og møbler ved sprang i $P(t)$.

h) $H_o(s) = H_r(s)\, H_p(s) \cdot H_m(s)$

$$= K_p \cdot \frac{1/h_v A_v}{\frac{mc}{h_v A_v} s + 1} \cdot \frac{1}{\frac{1}{0{,}01} s + 1} = \frac{t_o(s)}{n_o(s)}$$

$$M(s) = \frac{H_o(s)}{1 + H_o(s)} = \frac{\frac{t_o(s)}{n_o(s)}}{1 + \frac{t_o(s)}{n_o(s)}} = \frac{t_o(s)}{n_o(s) + t_o(s)}$$

$$= \frac{K_p / h_v A_v}{\left(\frac{mc}{h_v A_v} s + 1\right)\left(\frac{1}{0{,}01} s + 1\right) + \frac{K_p}{h_v A_v}}$$

$$N(s) = \frac{1}{1+H_0(s)} = \frac{1}{1+\frac{t_0(s)}{n_0(s)}} = \frac{n_0(s)}{n_0(s)+t_0(s)}$$

$$= \frac{\left(\frac{mc}{h_v A_v}s+1\right)\left(\frac{1}{0.01}s+1\right)}{\left(\frac{mc}{h_v A_v}s+1\right)\left(\frac{1}{0.01}s+1\right)+\frac{K_p}{h_v A_v}}$$

i) For å få 90^0 fasemargin må vi løfte H_0 - kurven m 23 dB

$$20\log(x) = 23$$

$$\underline{x = 14}$$

Ny $\underline{\underline{K_p}}$: $14 \times 0.001 = \underline{\underline{0.014}}$

Forsterkningsmarginen e uendelig

j) Anvender sluttverditeoremet på $e(s) = N(s) \cdot y_r(s)$ 11)

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot e(s)$$

$$= \lim_{s \to 0} s \cdot N(s) \cdot y_r(s) \qquad y_r(s) = \frac{1}{s}$$

$$= \lim_{s \to 0} \cancel{s} \cdot N(s) \cdot \frac{1}{\cancel{s}}$$

$$= N(0) = \frac{1}{1 + \frac{K_p}{h_v A_v}} = \frac{1}{1 + \frac{0.014}{0.002}} = \frac{1}{1+7} = \frac{1}{8}$$

Dess større K_p, dess mindre avvik, logisk

For å eliminere må regulator ha integral-virkning.

k) $\left.\begin{array}{l} m = 10 \\ c = 1 \\ h_r A_r = 0.002 \end{array}\right\}$ gir tidskonstant $T = \frac{mc}{h_r A_r} = \frac{10}{0.002}$

$= 5000$ sek

dvs 1.5 time

Ønsker reg.-sys dobbelt så raskt,

dvs $T_M = 2500$ sek.

$K = \frac{1}{h_r A_r} = \frac{1}{0.002} = 500$

Benytter pol/nullpkt kansellering:

$$K_p = \frac{5000}{2500 \cdot 500} = 0.004$$

$$T_i = 5000$$

Hvis vi velger polplassering: (ønsker ikke oversving $\zeta = 1$, $w_0 = \frac{1.5}{T_r} = \frac{1.5}{2500}$)

$$K_p = \frac{2 \cdot 1 \cdot \frac{1.5}{2500} \cdot 5000 - 1}{500}$$

$$= 0.01$$

$$T_i = \frac{2 \cdot 1 \cdot \frac{1.5}{2500} \cdot 5000 - 1}{\left(\frac{1.5}{2500}\right)^2 \cdot 5000} = 2777 \text{ sek}$$

Oppg 4)

A — 3

B — 5

C — 1

D — 2

E — 4