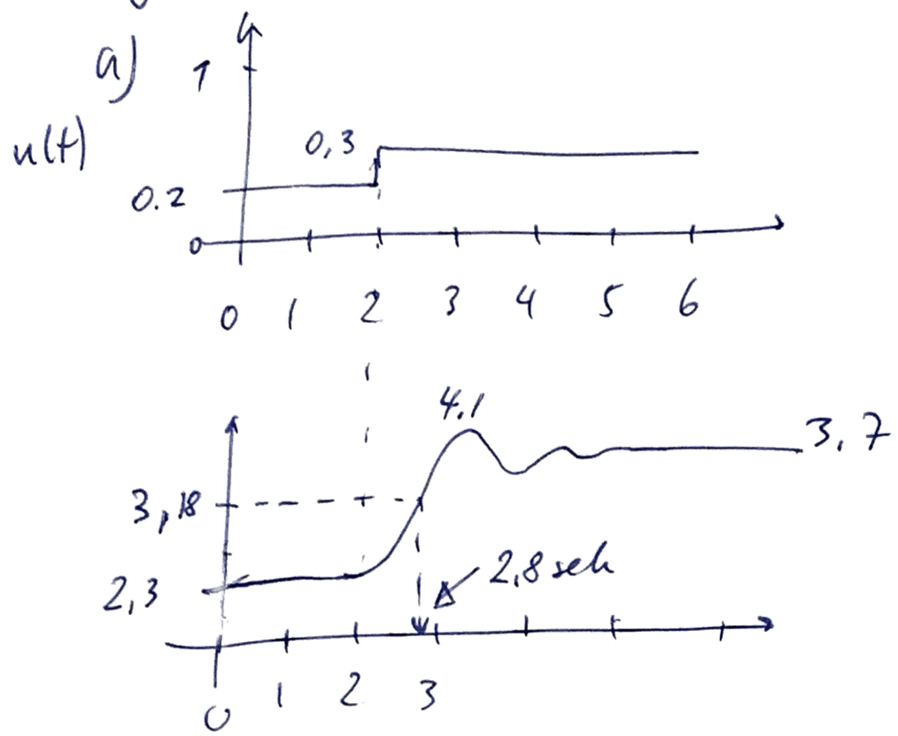
ELE320, eksamen 5. mai 2020

Hjemmeeksamen

Oppg 1.

a)



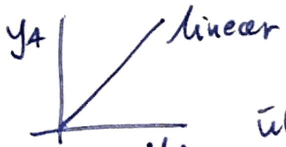
oppdiktet respons

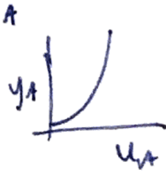
$u_A = 0.2$ $\qquad y_{max} = 4.1$ $\qquad \Delta u(t) = 0.1$

$y_A = 2,3$ $\qquad y_{ss} = 3.7$

b) Kan ikke si noe om prosessen er ulineær ut fra én enkelt sprangrespons.

Måtte gjort flere like sprang $\Delta u(t) = 0.1$ ved forskjellige arb.pkt u_A.





Dersom ulik respons i $y(t)$, ⇒ <u>ulineært</u>

ulineær

dvs ulik K, ω_0, ξ

c) $K = \frac{\Delta y}{\Delta u} = \frac{3.7-2.3}{0.3-0.2} = \frac{1.4}{0.1} = \underline{14}$ (2)

Tidspunkt for avlesing av T_r:

$\Delta y = 1.4$

63% av dette er 0.8820

Skal da avlese tidspunktet hvor $y(t)$ passerer $2.3 + 0.88 = 3.18$

Dette betyr at $T_r = 2{,}8 - 2 = 0{,}8$ sek

$$\boxed{w_0 = \frac{1.5}{T_r} = \underline{\underline{1.875}}}$$

Vil også godkjenne $w_0 = \frac{1}{T_r} = 1.25$

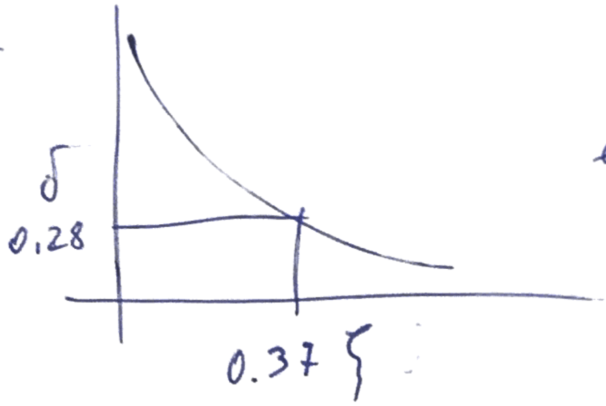
Oversvingsfaktor $\delta = \frac{y_{max} - y_{ss}}{y_{ss}}$

$$= \frac{(4.1-2.3)(3{,}7-2.3)}{(3.7-2.3)}$$

$$= 0.2857$$

$\delta = \underline{\underline{28\%}}$

Ut fra



~~avles~~ avleses $\xi = 0.37$

Finner da

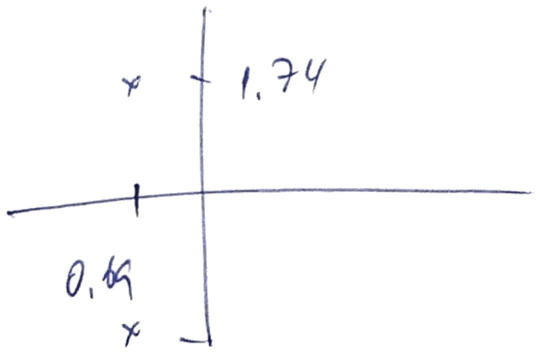
$$H_p(s) = \frac{14}{\frac{1}{(1.875)^2} s^2 + 2 \cdot \frac{0.37}{1.875} s + 1}$$

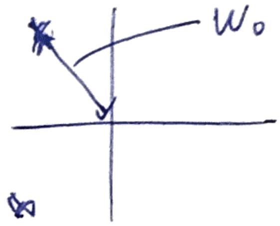
$$= \frac{14}{0.284 s^2 + 0.394 s + 1}$$

d) $0.284 s^2 + 0.394 s + 1 = 0$

$$s_{1,2} = \frac{-0.394 \pm \sqrt{0.394^2 - 4 \cdot 0.284}}{2 \cdot 0.284}$$

$$= -0.69 \pm j\,1.74$$





e) Avstanden til polene representerer w_0. Dette kan vises ved å se på poluttrykket

$$s^2 + 2\xi w_0 + w_0^2 = 0$$

$$s_{1,2} = \frac{-2\xi w_0 \pm \sqrt{(2\xi w_0)^2 - 4 \cdot w_0^2}}{2}$$

$$= \frac{-2\xi w_0 \pm \sqrt{4\xi^2 w_0^2 - 4 w_0^2}}{2}$$

$$= \frac{-2\xi w_0 \pm 2 w_0 \sqrt{\xi^2 - 1}}{2}$$

$$= -\xi w_0 \pm w_0 \sqrt{1-\xi^2}$$

$$= -\xi w_0 \pm w_0 j \sqrt{1-\xi^2}$$

Avstanden til polene er:

$$|Z| = \sqrt{Re^2 + Im^2}$$

$$= \sqrt{(-\xi w_0)^2 + (w_0(\sqrt{1-\xi^2}))^2}$$

$$= \sqrt{\xi^2 w_0^2 + w_0^2 (1-\xi^2)}$$

$$= \sqrt{w_0^2}$$

$$= \underline{\underline{w_0}}$$

$$= \sqrt{(0.69)^2 + (1.74)^2}$$

$$= \sqrt{0.476 + 3.02}$$

$$= \sqrt{3.50}$$

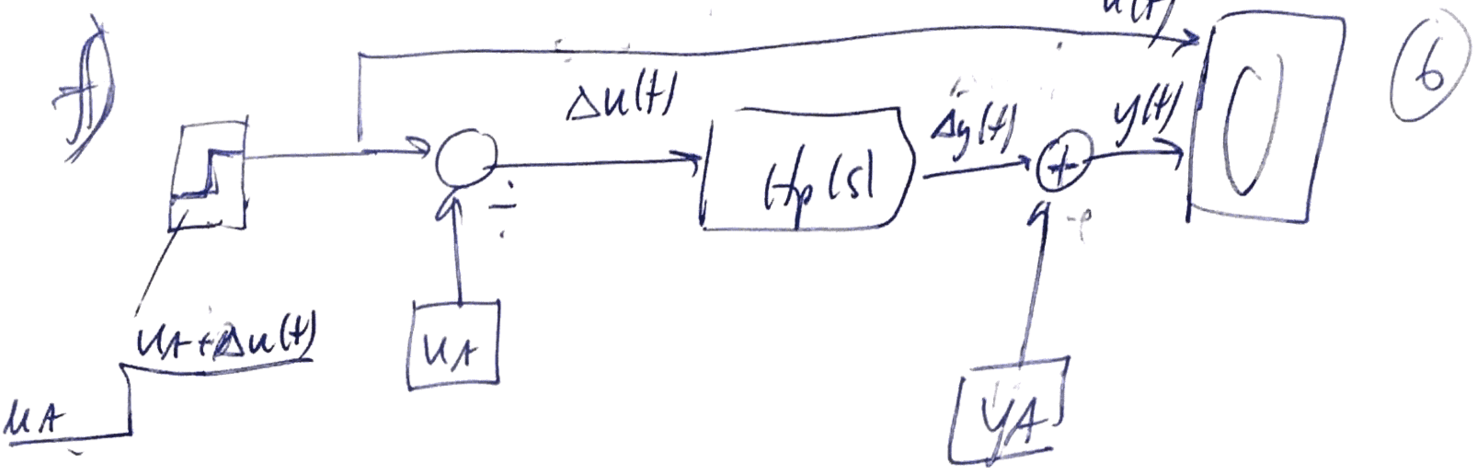
$$= \underline{\underline{1.87}}$$

$$w_0 = 1.87$$

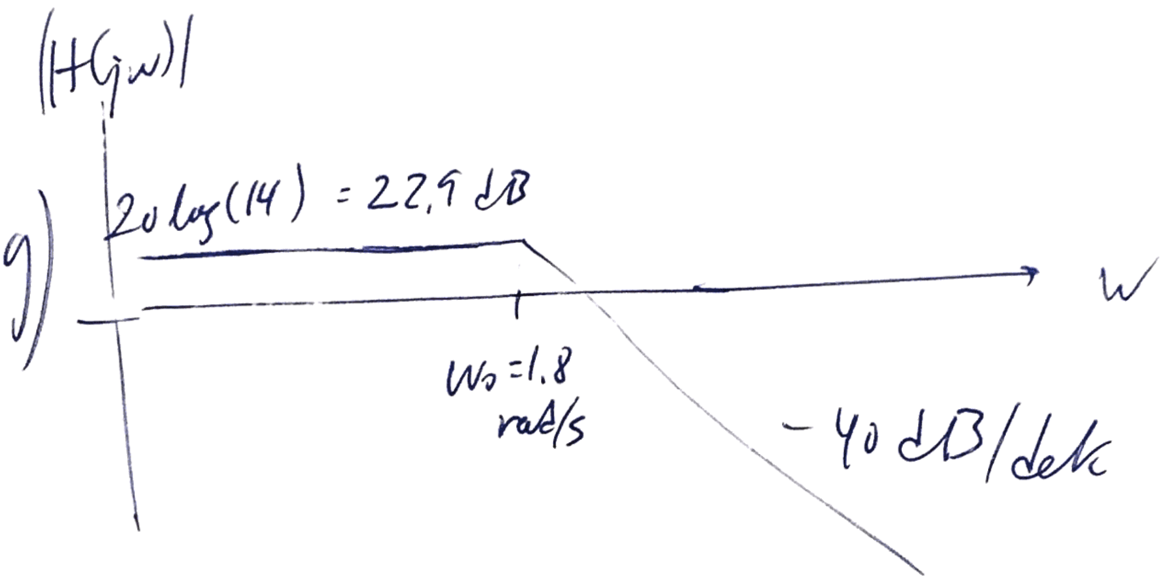
Derfor: $T_r = \frac{1.5}{w_0} = \frac{1.5}{1.875}$

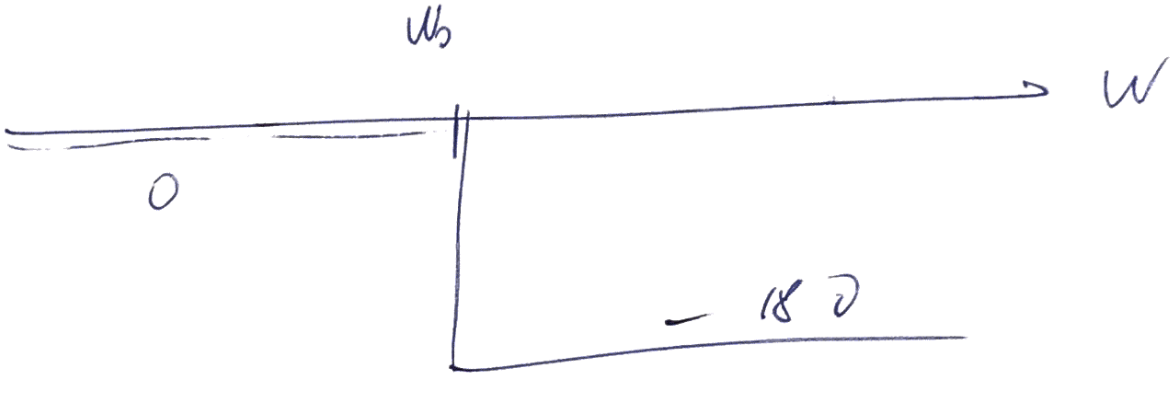
$$= \underline{\underline{0.8 \text{ sek}}}$$

f)



g)



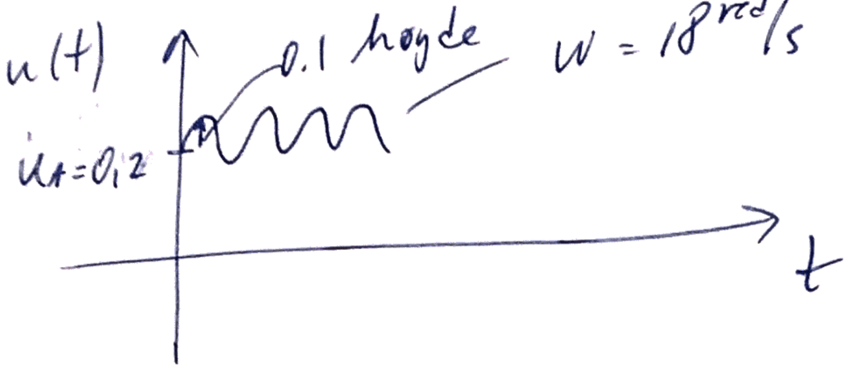


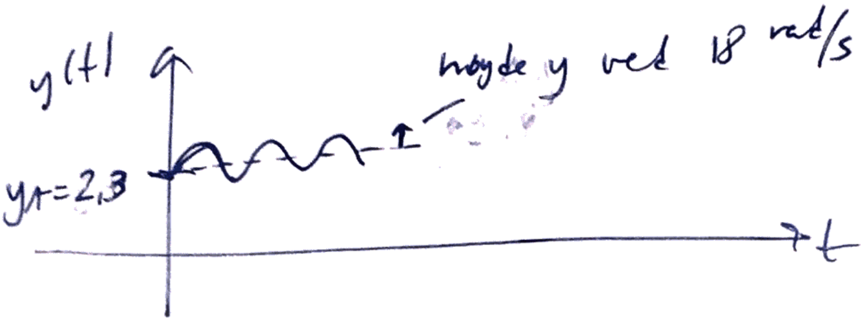
Her ser vi ikke uA/yA.

Bare oppførsel omkring arb.pkt.

h) Prinsipiell skisse

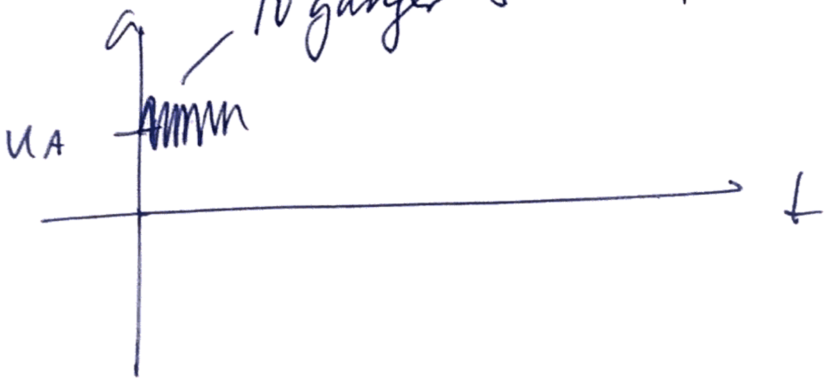
$w = 18$ rad/s (som eksempel)





$w = 180$ rad/s (en dekade opp)

10 ganger så rask, samme amplitude



amplitude er 0.01 så stor som for $w = 18$ rad/s

høyde y ved 180 rad/s $= 0.01 \cdot$ y ved 18 rad/s

i) $$H_p(s) = \frac{14}{0.284 s^2 + 0.394 s + 1}$$ (8)

$s = j\omega$

$$H(j\omega) = \frac{14}{-0.284 \omega^2 + 0.394 j\omega + 1}$$

$$= \frac{14}{1 - 0.284 \omega^2 + j\,0.394 \omega}$$

$$= \frac{14}{\sqrt{(1-0.284\omega^2)^2 + (0.394\omega)^2}} \cdot e^{j\left(-\operatorname{atan}\left(\frac{0.394\omega}{1-0.284\omega^2}\right)\right)}$$

$$\text{hwor } |H(j\omega)| = \frac{14}{\sqrt{(1-0.284\omega^2)^2 + (0.394\omega)^2}}$$

$$\angle H(j\omega) = -\operatorname{atan}\left(\frac{0.394\omega}{1-0.284\omega^2}\right)$$

j) s = tf('s')

hp = 14 / (0.284 * s^2 + 0.394 * s + 1)

bode(hp)

velger $w = 1.21$ rad/s fra figuren.

Avlest $|H(jw)| = 25.4$ dB

$\angle H(jw) = -39.3°$

k) $w = 1.21$ innsatt i

$$|H(jw)| = \frac{14}{\sqrt{(1 - 0.284 \cdot 1.21^2)^2 + (0.394 \cdot 1.21)^2}}$$

$$= \frac{14}{\sqrt{0.3413^2 + 0.2273}}$$

$$= \frac{14}{0.754} = 18{,}56$$

$$|H(j\omega)|_{dB} = 20\log(18{,}56)$$

$$= 25{,}37\,dB$$ (Stemmer med avlest)

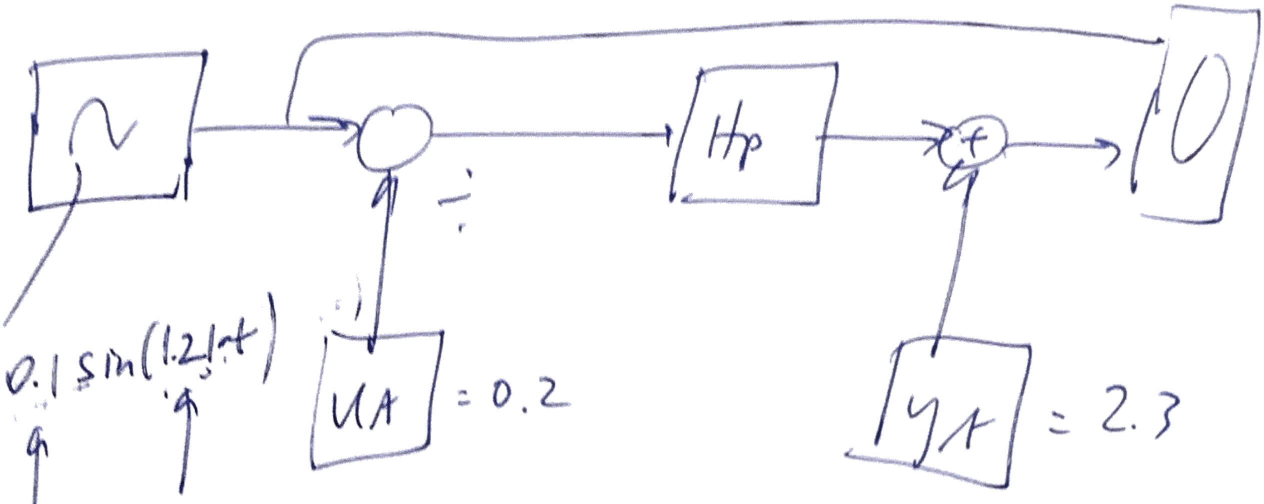
$$\angle H(j\omega) = -\text{atan}\left(\frac{0{,}394 \cdot 1.21}{1 - 0{,}28 \cdot 1.21^2}\right)$$

$$= -\text{atan}(0{,}8080)$$

$$= \underline{\underline{-38{,}9^\circ}}$$

<u>Stemmer bra</u> !!

e)

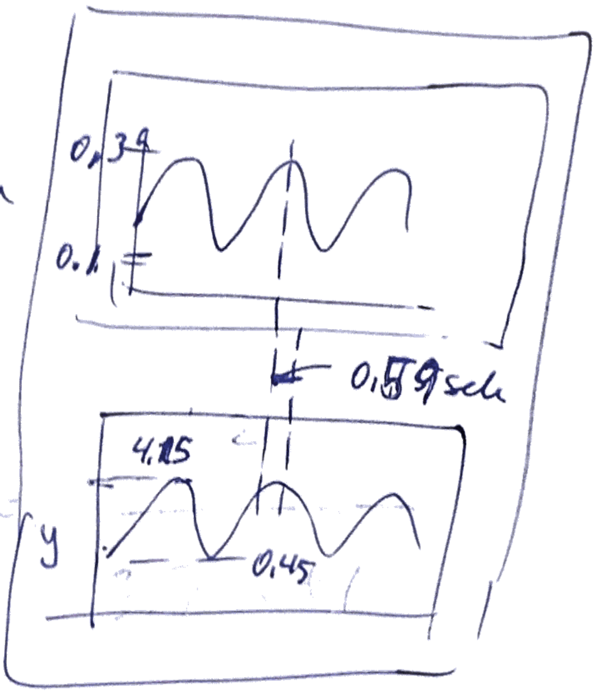


$u(t) = u_A + 0.1 \sin(1.21 \cdot t)$

bias = 0.2

ampl. = 0.1

frekvens = 1.21



~~Metode~~	~~Kp~~	~~Ti~~
~~Polplassering~~		
~~Pol/nullpunktkans..~~		
~~Stigetid~~		

$\Delta u = 0.1$

$$\Delta y = \frac{4.15 - 0.45}{2} = 1.85$$

$$|H(j\omega_1)| = \frac{1.85}{0.1} = 18.5$$

$$|H(j\omega_1)|_{dB} = 25.34 \text{ dB}$$ stemme bra

$\omega_1 = 1.21$ rad/s

$\omega = 2\pi f \Rightarrow f = \frac{1}{T_p}$

$$T_p = \frac{2\pi}{\omega} = \frac{6.28}{1.21} = 5.19 \text{ sek.}$$

$$\frac{\Delta t}{T_p} = \frac{\varphi}{360}$$

$$\frac{0.59}{5.19} = \frac{\varphi}{360} \Rightarrow \underline{\underline{\varphi = 40.9^\circ}}$$

stemme bra

Oppg 2 $K = 3.7, \quad T = 4.5$

a) Polplassering: Ønsker $T_r = \frac{4.5}{3} = 1.5$ sekund

$$\Rightarrow \omega_0 = \frac{1.5}{T_r} = 1$$

Ønsker 10% oversving

$$\delta = 0.1 \Rightarrow \zeta = 0.6$$

$$K_p = \frac{2 \cdot 0.6 \cdot 1 \cdot 4.5 - 1}{3.7} = 1.18$$

$$T_i = \frac{2 \cdot 0.6 \cdot 1 \cdot 4.5 - 1}{1^2 \cdot 4.5} = 0.97$$

Pol/nullpkt kansellering: $T_i = T = 4.5$

Spesifiserer $T_M = 1.5$ sek. $\left(\frac{4.5}{3}\right)$

$$K_p = \frac{4.5}{1.5 \cdot 3.7} = 0.81$$

$$\underline{\underline{T_i = 4.5}}$$

Skogestad: $T_c = 1.5$, $k_1 = 1.4$

$$K_p = \frac{4.5}{1.5 \cdot 3.7} = 0.81$$

$$T_i = 2.1$$

	K_p	T_i
Polplassering	1.18	0.97
Pol/nullpunkt	0.81	4.5
Skogestad	0.81	2.1

b) Anta $H_m(s) = 1$.

$$H_0(s) = H_r(s) \cdot H_p(s)$$

$$H_r(s) = \frac{K_p(T_i s + 1)}{T_i s}$$

$$M_1(s) = \frac{\frac{K_p(T_i s+1)}{T_i s} \cdot \frac{K}{Ts+1}}{1 + \frac{K_p(T_i s+1)}{T_i s} \cdot \frac{K}{Ts+1}}$$

$$= \frac{K_p(T_i s+1) \cdot K}{T_i s(Ts+1) + K_p(T_i s+1) \cdot K}$$

$$= \frac{K K_p \cdot K(T_i s+1) K}{T_i \cdot T s^2 + (T_i + K K_p T_i) s + K_p K}$$

(15)

$$M(s) = \frac{T_i s + 1}{\frac{T_i \cdot T}{K_p \cdot K} s^2 + \frac{T_i + K_p T_i K}{K_p \cdot K} s + 1}$$

$$M_1(s) = \frac{0.97 s + 1}{\frac{0.97 \cdot 4.5}{1.18 \cdot 3.7} s^2 + \frac{0.97 + 1.18 \cdot 0.97 \cdot 3.7}{1.18 \cdot 3.7} s + 1}$$

(polplassering)

$$= \frac{0.97 s + 1}{s^2 + 1{,}194 s + 1}$$

$M_2(s) = ?$

pol/nullpkt kansellering

$$H_0(s) = \frac{K_p \cancel{(T_i s + 1)}}{T_i s} \cdot \frac{K}{\cancel{T s + 1}}$$

$$= \frac{K_p K}{T_i s}$$

$$M(s) = \frac{\frac{K_p K}{T_i s}}{1 + \frac{K_p K}{T_i s}} = \frac{1}{\frac{T_i}{K_p K} s + 1}$$

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$$M_2(s) = \frac{1}{\frac{4.5}{0.81 \cdot 3.7} s + 1} = \frac{1}{1.5 s + 1}$$

$\downarrow$

ikke oversvingende

$$\underline{\underline{T_m = 1.5}}$$

Skogestad (brukes samme M(s) som for polplassering)

$$M_3(s) = \frac{T_i s + 1}{\frac{T_i T}{k_p k} s^2 + \frac{T_i + k_p T_i k}{k_p k} s + 1}$$

$$= \frac{2.1 s + 1}{\frac{2.1 \cdot 4.5}{0.81 \cdot 3.7} s^2 + \frac{2.1 + 0.81 \cdot 2.1 \cdot 3.7}{0.81 \cdot 3.7} s + 1}$$

$$= \frac{2.1 s + 1}{3.15 s^2 + 2{,}8 s + 1}$$

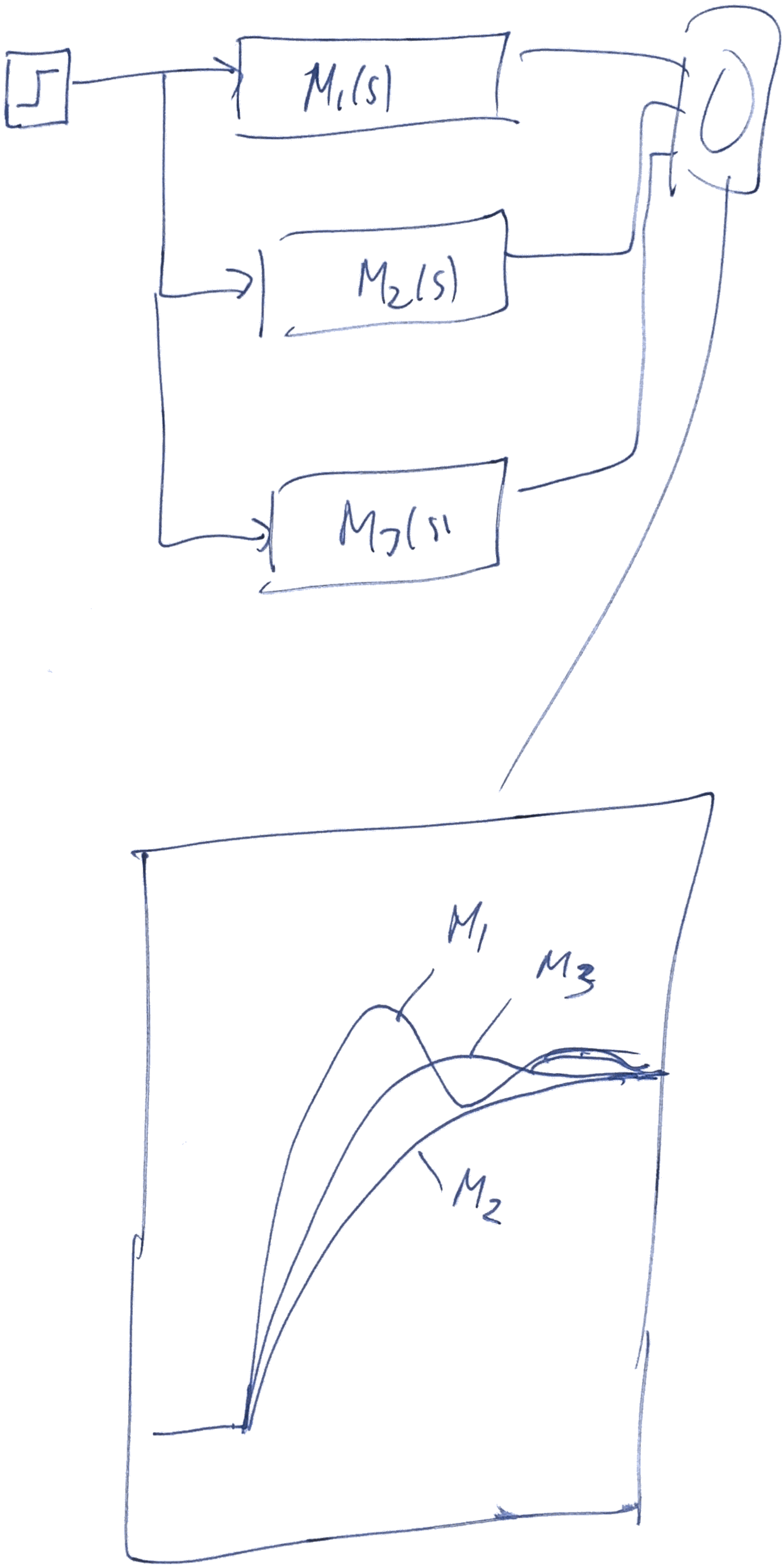
skal ligne på

$$M(s) = \frac{1}{1{,}5 s + 1}$$

$\uparrow$

T_m

4)
c)



18 2

d)

$$M_1(s) = \frac{0,97s+1}{s^2+1.19s+1}$$

Nullpkt $0,97s+1=0$

$\Downarrow$

$s = -\frac{1}{0.97}$

$= -1.03$

poler:

$s^2+1.19s+1=0$

$$s_{1,2} = \frac{-1.19 \pm \sqrt{1,19^2-4}}{2}$$

$= -0,595 \pm j0.8037$

$$M_2(s) = \frac{1}{1.5s+1}$$

pol: $1.5s+1=0$

$s = -\frac{1}{1.5}$

$= -0.66$

$$M_3(s) = \frac{2.1s+1}{3.15s^2+2,8s+1}$$

Nullpkt $2.1s+1=0$

$s = -\frac{1}{2.1} =$

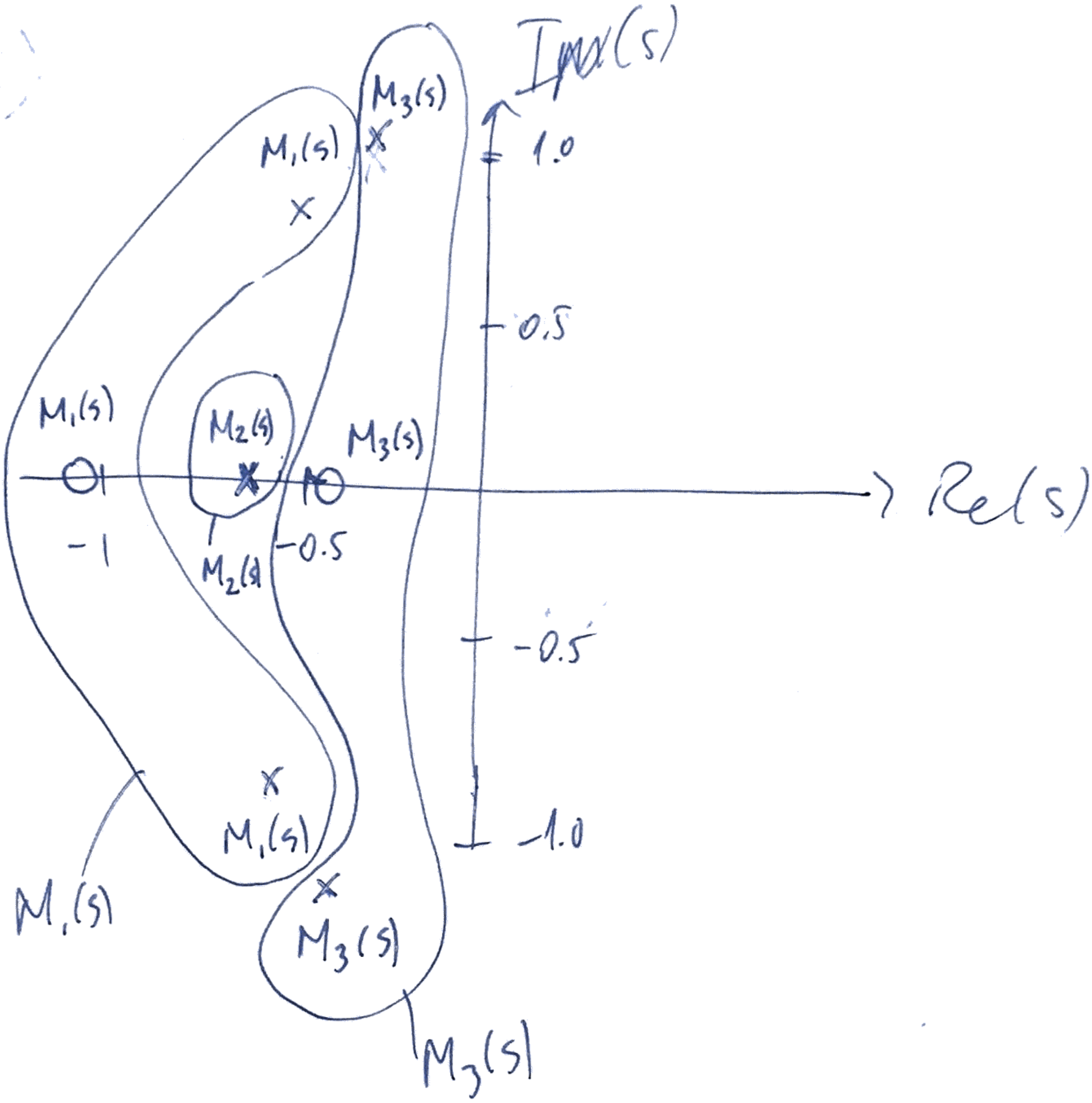
$= -0.47$

poler:

$3,15\,s^2+2,8\,s+1=0$

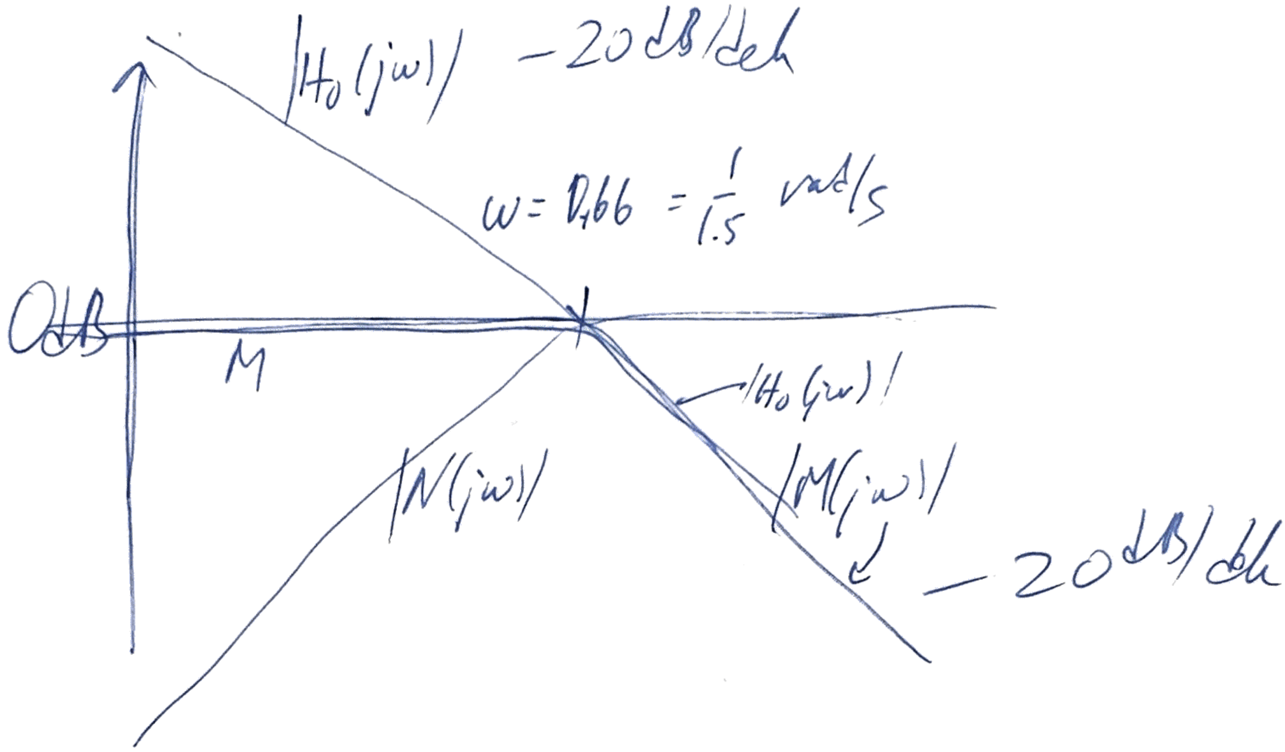
$$s_{1/2} = \frac{-2,8 \pm \sqrt{2,8^2-4\cdot 3.15}}{2\cdot 3,15}$$

$M_3(s)$: $s_{1,2} = -0.44 \pm j\,1.09$



Systemet med størst oversving er $M_1(s)$, men $M_3(s)$ har poler med størst imaginære bidrag.
Årsaken til $M_1(s)$ sitt oversving er bidraget fra nullpunktet til $M_1(s)$.
Bidraget fra $M_3(s)$ sitt nullpkt er mindre.

f) Velger det enkleste $M_2(s)$ (20)



e)

```
s = tf('s')
H_p = 3,7/(4.5*s+1)
H_r_1 = 1,18*(0.97*s+1)/(0.97*s)
H_0_1 = Hp * Hr_1;

H_r_2 = 0,81*(4.5*s+1)/(4,5*s)

H_0_2 = Hp * H_r_2;
Hr_3 = 0,81*(2.1*s+1)/(2.1*s)
H_0_3 = H_p * H_r_3;
```

```
margin(H_0_1)
hold on
margin(H_0_2)
margin(H_0_3)
```

$M_1(s)$: $\Delta K = \infty$, $\varphi = 60.4$

$M_2(s)$: $\Delta K = \infty$, $\varphi = 90$

$M_3(s)$: $\Delta K = \infty$, $\varphi = 74.2$

Vi ser at dess mindre φ, dess mer oversving.