Formelsamling for ELE100 Elektroteknikk 1

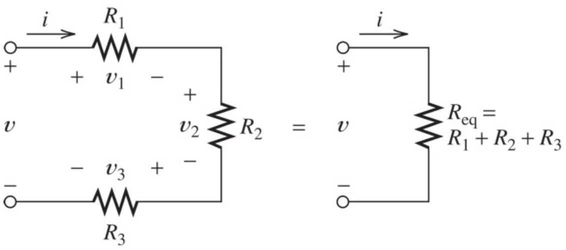
$$i(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} \qquad q(t) = \int_{t_0}^{t} i(t)\,\mathrm{d}t + q(t_0) \tag{1.1}$$

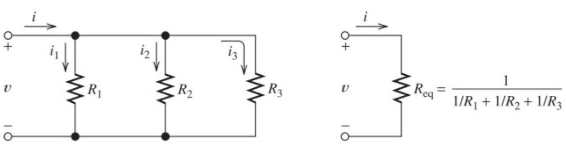
$$p = vi \qquad p(t) = v(t)i(t) \qquad w = \int_{t_0}^{t_1} p(t)\,\mathrm{d}t \tag{1.2}$$

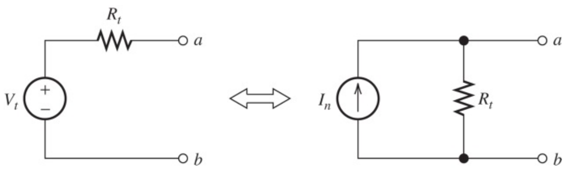
$$i = \frac{v}{R} = \frac{1}{R}v = Gv \tag{1.3}$$

$$R = \rho\frac{l}{A} = \frac{1}{\sigma}\frac{l}{A} \tag{1.4}$$

$$\sum_{\text{alle } k} i_k = 0 \qquad \sum_{\text{alle } k} v_k = 0 \tag{1.5}$$

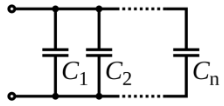




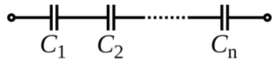


$$q = Cv \qquad i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathrm{d}(Cv)}{\mathrm{d}t} = C\frac{\mathrm{d}v}{\mathrm{d}t} \tag{1.6}$$

$$w(t) = \frac{1}{2}Cv^2(t) \qquad w = \frac{1}{2}v(t)q(t) = \frac{q^2(t)}{2C} \tag{1.7}$$

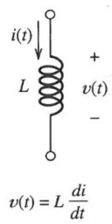


$$C_{eq} = (C_1 + C_2 + \cdots + C_n) \tag{1.8}$$

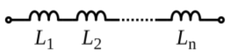


$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)^{-1} \tag{1.9}$$

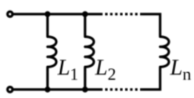
$$C = \frac{\varepsilon A}{d} \qquad \varepsilon = \varepsilon_r\varepsilon_0 \quad \text{hvor} \quad \varepsilon_0 = 8.854 \cdot 10^{-12}\ \mathrm{F/m} \tag{1.10}$$



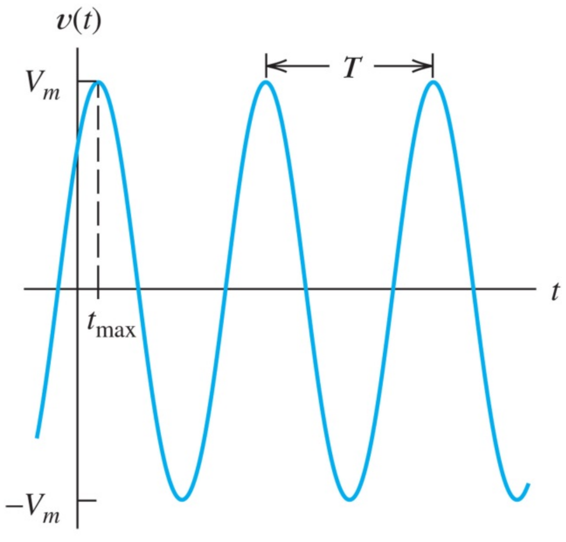
$$p(t) = v(t)i(t) = Li(t)\frac{\mathrm{d}i}{\mathrm{d}t} \qquad w(t) = \frac{1}{2}Li^2(t) \tag{1.11}$$



$$L_{eq} = (L_1 + L_2 + \cdots + L_n) \tag{1.12}$$



$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}\right)^{-1} \tag{1.13}$$



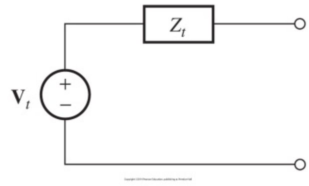
$$v(t) = V_m \cos(\omega t + \theta)$$
$$\omega T = 2\pi \quad \Leftrightarrow \quad \omega = \frac{2\pi}{T} \qquad (1.14)$$
$$f = \frac{1}{T} \quad \Rightarrow \quad \omega = 2\pi f$$

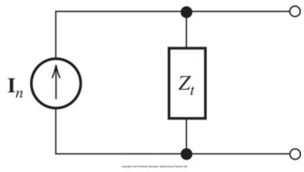
$$E_T = \int_0^T p(t)\,\mathrm{d}t$$
$$P_{middel} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t$$
$$P_{middel} = \frac{V_{rms}^2}{R} = I_{rms}^2 R \qquad (1.15)$$
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,\mathrm{d}t}$$
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,\mathrm{d}t}$$

$$v_1(t) = V_1 \cos(\omega t + \theta_1)$$
$$\mathbf{V}_1 = V_1 \angle \theta_1$$
$$e^{j\theta} = \cos\theta + j \cdot \sin\theta \qquad (1.16)$$
$$\cos\theta = \mathrm{Re}\left\{e^{j\theta}\right\}$$
$$v_1(t) = V_1 \cos(\omega t + \theta_1) = \mathrm{Re}\left\{V_1 e^{j(\omega t + \theta_1)}\right\}$$
$$\mathbf{V}_1 = V_1 \angle \theta_1$$

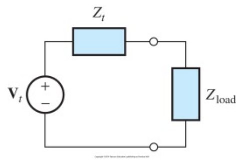
$$P = V_{rms} I_{rms} \cos(\theta)$$
$$Q = V_{rms} I_{rms} \sin(\theta) \qquad (1.17)$$
$$S = V_{rms} I_{rms}$$

$$\mathbf{I} = I_m \angle \theta_i \qquad \mathbf{V} = V_m \angle \theta_v$$
$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* \qquad (1.18)$$
$$= P + jQ$$





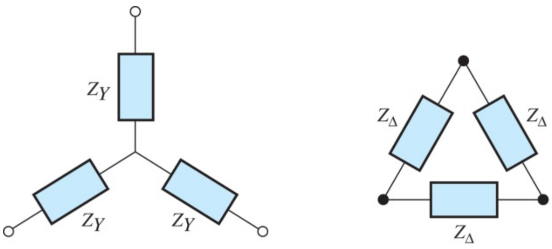
$$\mathbf{V}_t = \mathbf{V}_{oc} \qquad \mathbf{I}_n = \mathbf{I}_{sc}$$
$$\mathbf{Z}_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{V}_t}{\mathbf{I}_{sc}} \qquad (1.19)$$



$$\mathbf{Z}_{load} = \mathbf{Z}_t^*$$
$$\mathbf{Z}_t = R_t + jX_t \qquad (1.20)$$
$$\mathbf{Z}_{load} = \mathbf{Z}_t^* = R_t - jX_t$$

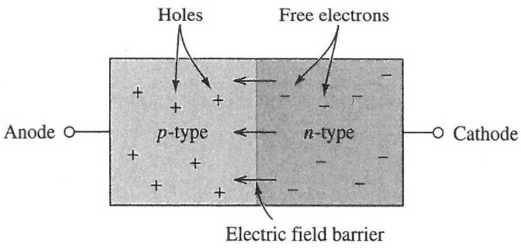
$$v_{an}(t) = V_Y \cos(\omega t)$$
$$v_{bn}(t) = V_Y \cos(\omega t - 120°)$$
$$v_{cn}(t) = V_Y \cos(\omega t + 120°) \qquad (1.21)$$
$$\mathbf{V}_{an} = V_Y \angle 0°$$
$$\mathbf{V}_{bn} = V_Y \angle -120°$$
$$\mathbf{V}_{cn} = V_Y \angle +120°$$

$$P = \sqrt{3} V_{linje} I_{linje} \cos(\theta) \qquad (1.22)$$



(a) Wye-connected load (b) Delta-connected load

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad (1.23)$$



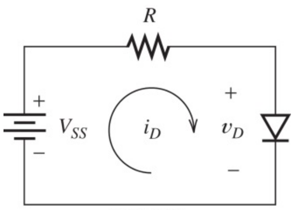
(c) Simplified physical structure

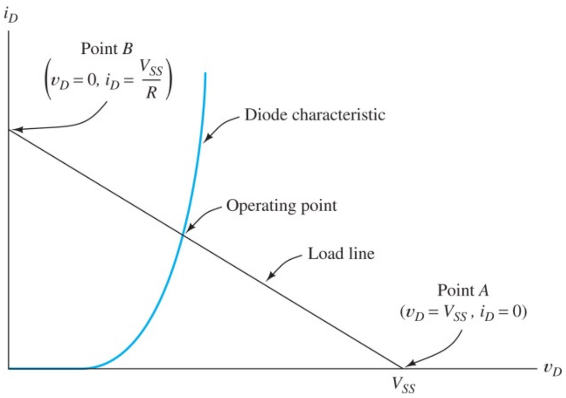
$$i_D = I_S\left[e^{\frac{v_D}{nV_T}} - 1\right]$$

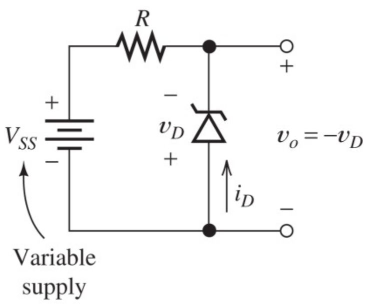
$$V_T = \frac{kT}{q} \qquad (1.24)$$

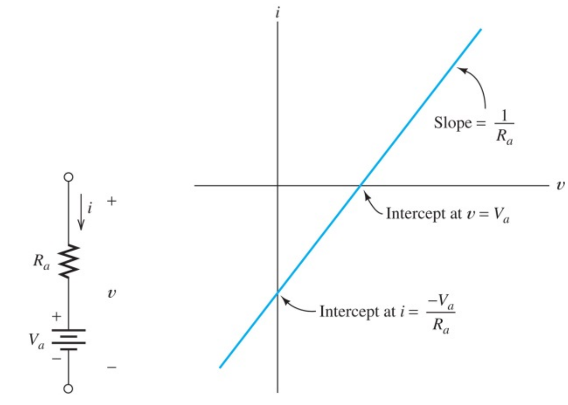
$$k = 1.38 \cdot 10^{-23}\ \text{J/K}$$

$$q = 1.60 \cdot 10^{-19}\ \text{C}$$

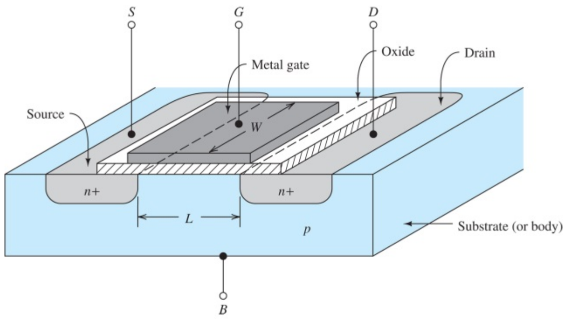








(a) Circuit diagram (b) Volt–ampere characteristic



$$i_D = 0 \quad \text{for} \quad v_{GS} \le V_{t0}$$

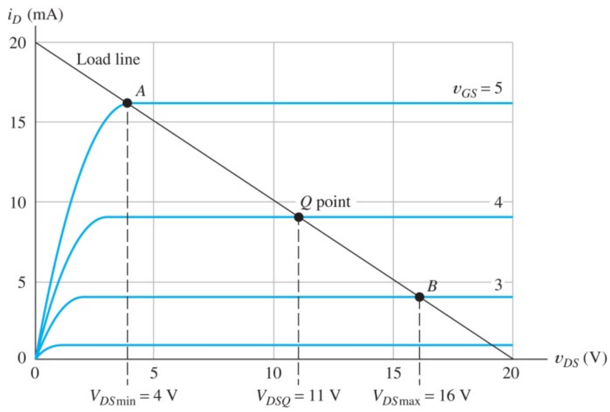
$$v_{GS} \ge V_{t0} \quad \text{og} \quad v_{DS} < v_{GS} - V_{t0}$$

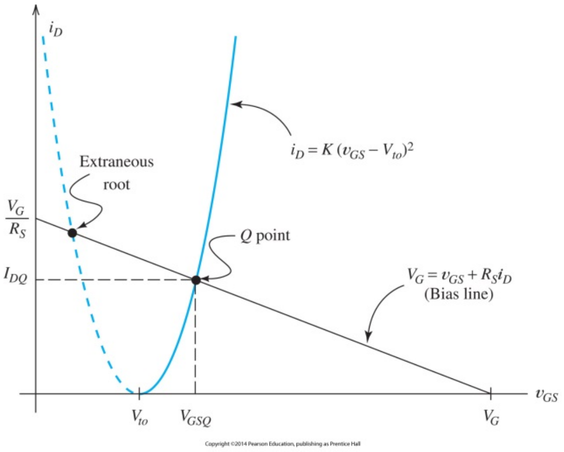
$$i_D = K\left[2\left(v_{GS} - V_{t0}\right)v_{DS} - v_{DS}^2\right] \qquad (1.25)$$

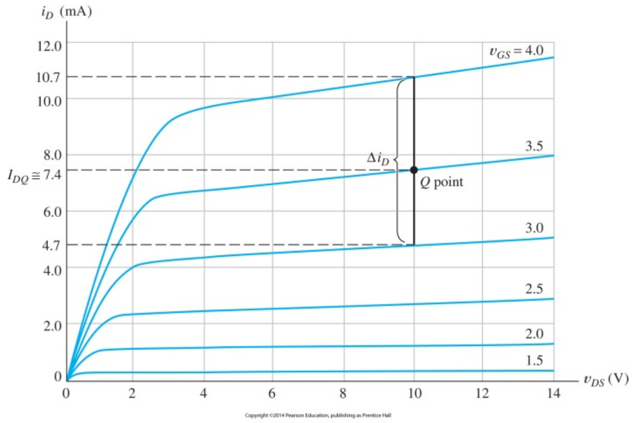
$$K = \left(\frac{W}{L}\right)\frac{KP}{2}$$

$$v_{GS} \ge V_{t0} \quad \text{og} \quad v_{DS} \ge v_{GS} - V_{t0}$$

$$i_D = K\left(v_{GS} - V_{t0}\right)^2 \qquad (1.26)$$



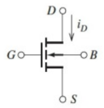
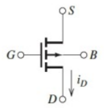


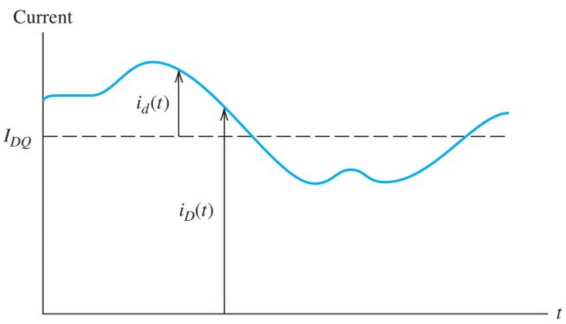


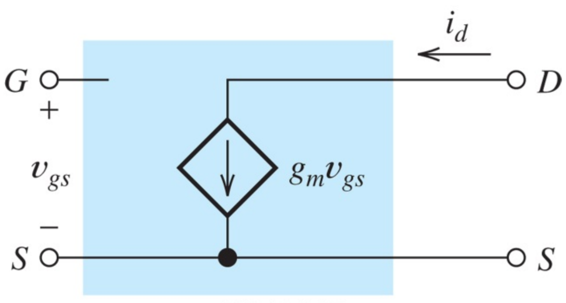
$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Arbeidspunkt} \cong \left.\frac{\Delta i_D}{\Delta v_{GS}}\right|_{v_{DS}=V_{DSQ}} \qquad (1.28)$$

$$\frac{1}{r_d} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Arbeidspunkt} \cong \left.\frac{\Delta i_D}{\Delta v_{DS}}\right|_{v_{GS}=V_{GSQ}}$$

Table 12.1. MOSFET Summary

	NMOS	PMOS
Circuit symbol		
KP (typical value)	50 μA/V^2	25 μA/V^2
K	(1/2) KP (W/L)	(1/2) KP (W/L)
V_{to} (typical value)	+1 V	−1 V
Cutoff region	$v_{GS} \le V_{to}$ $i_D = 0$	$v_{GS} \ge V_{to}$ $i_D = 0$
Triode region	$v_{GS} \ge V_{to}$ and $0 \le v_{DS} \le v_{GS} - V_{to}$ $i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$	$v_{GS} \le V_{to}$ and $0 \ge v_{DS} \ge v_{GS} - V_{to}$ $i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2]$
Saturation region	$v_{GS} \ge V_{to}$ and $v_{DS} \ge v_{GS} - V_{to}$ $i_D = K(v_{GS} - V_{to})^2$	$v_{GS} \le V_{to}$ and $v_{DS} \le v_{GS} - V_{to}$ $i_D = K(v_{GS} - V_{to})^2$
v_{DS} and v_{GS}	Normally assume positive values	Normally assume negative values

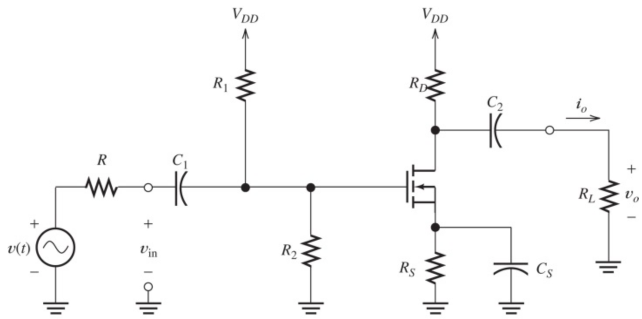




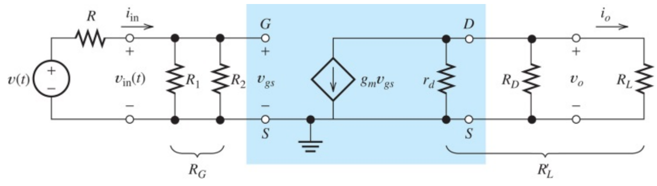
$$g_m = 2\sqrt{KI_{DQ}}$$

$$g_m = \sqrt{2KP}\sqrt{\frac{W}{L}}\sqrt{I_{DQ}} \qquad (1.27)$$

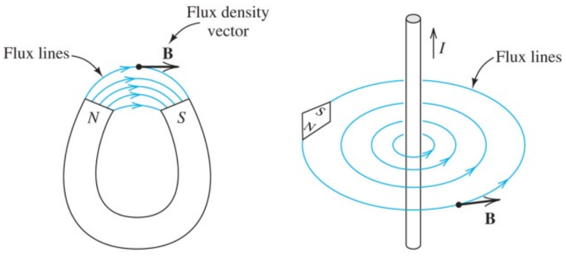
Transistor-forsterker:



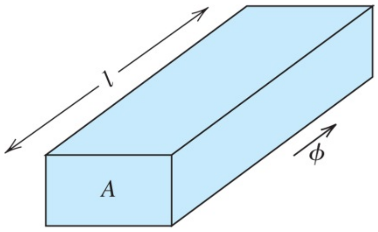
Figur 12.22



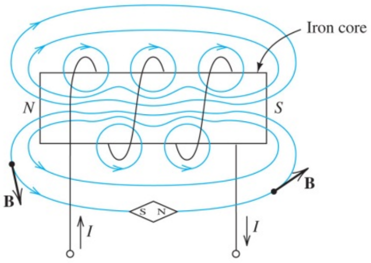
Figur 12.23



(a) Permanent magnet (b) Field around a straight wire carrying current I



$\mathcal{R} = \frac{l}{\mu A}$



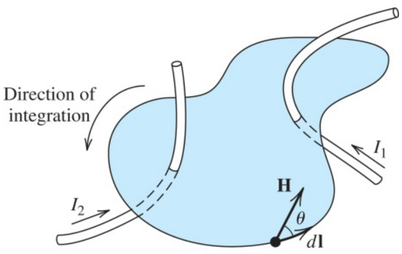
(c) Field for a coil of wire

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} \quad \text{hvor}$$
$$\mu_0 = 4\pi 10^{-7}\,\text{Vs/Am} \qquad (1.29)$$

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B}$$
$$|\mathbf{f}| = q|\mathbf{u}||\mathbf{B}|\sin(\vartheta)$$
$$d\mathbf{F} = Id\mathbf{s} \times \mathbf{B} \qquad (1.30)$$
$$F = BIl$$

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{A} \quad \text{eller}$$
$$\Phi = BA\cos(\beta) \qquad (1.31)$$
$$\lambda = N\Phi$$

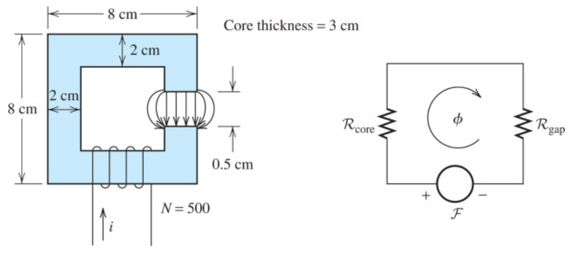
$$e = \frac{d\lambda}{dt}$$
$$\lambda = Bldx \qquad (1.32)$$
$$e = \frac{d\lambda}{dt} = Bl\frac{dx}{dt} = Blv$$



$\oint \mathbf{H} \bullet d\mathbf{l} = I_1 + I_2$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \oint H\cos(\theta)\,dl = \sum I_k \qquad (1.33)$$

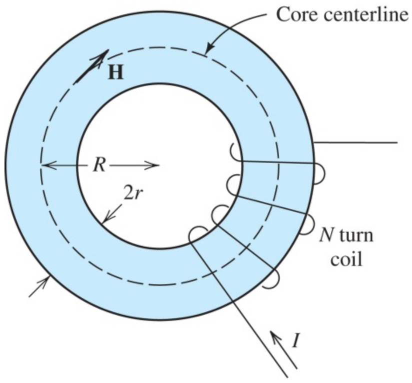
$$\mathcal{R} = \frac{l}{\mu A}$$
$$\mathcal{F} = NI \qquad (1.34)$$
$$\mathcal{F} = \mathcal{R}\Phi$$



(a) Iron core with an air gap (b) Magnetic circuit

Som **håndregel** øker vi dimensjonen i luftgapets tverrsnitt med lengden av luftgapet i begge retninger.

Toroid spole:



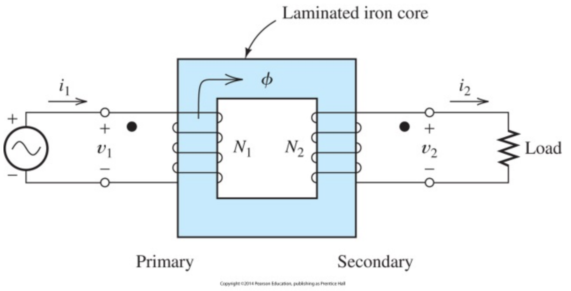
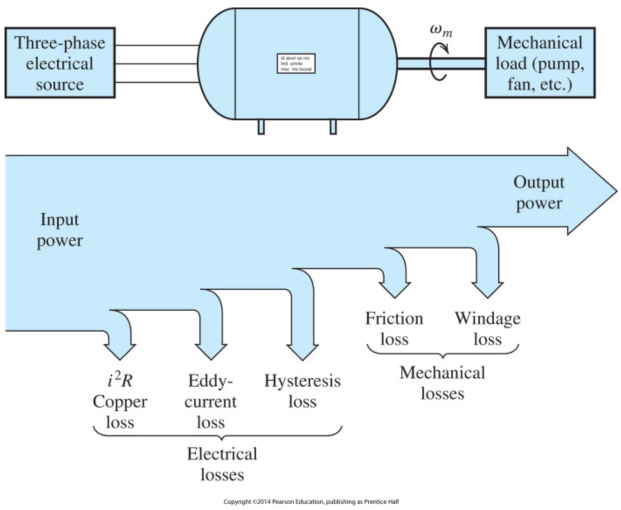


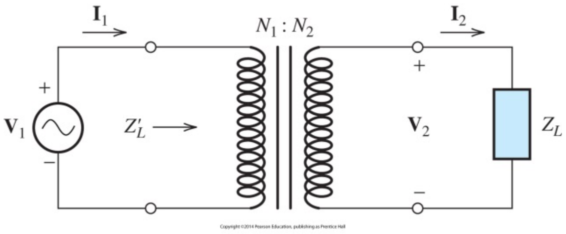
Table 15.1. **Circuit Values of a 60-Hz 20-kVA 2400/240-V Transformer Compared with Those of an Ideal Transformer**

Element Name	*Symbol*	*Ideal*	*Real*
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

$$L = \frac{\lambda}{i} \quad , \quad L_1 = \frac{\lambda_{11}}{i_1} \quad L_2 = \frac{\lambda_{22}}{i_2} \qquad (1.35)$$
$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

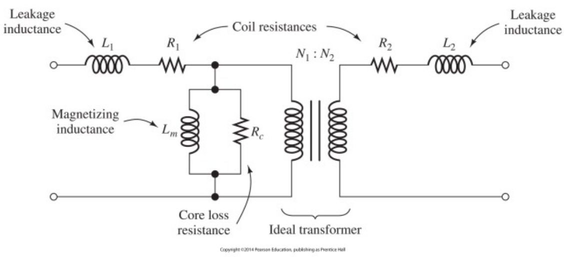


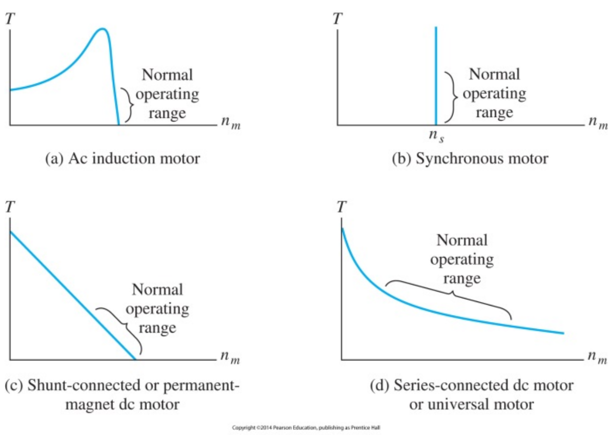
$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$
$$N_1 i_1(t) - N_2 i_2(t) = 0 \quad \Rightarrow \qquad (1.36)$$
$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$



$$P_{ut} = T_{ut}\omega_m$$
$$\omega_m = n_m \cdot \frac{2\pi}{60}$$
$$P_{hp} = \frac{P_{watt}}{745.7} \qquad (1.38)$$
$$\eta = \frac{P_{ut}}{P_{inn}} \cdot 100\%$$

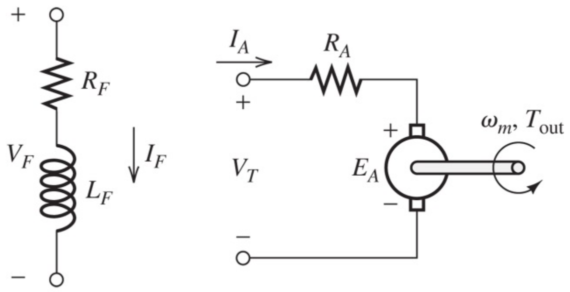
$$\mathbf{Z}_L^{'} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \mathbf{Z}_L \qquad (1.37)$$



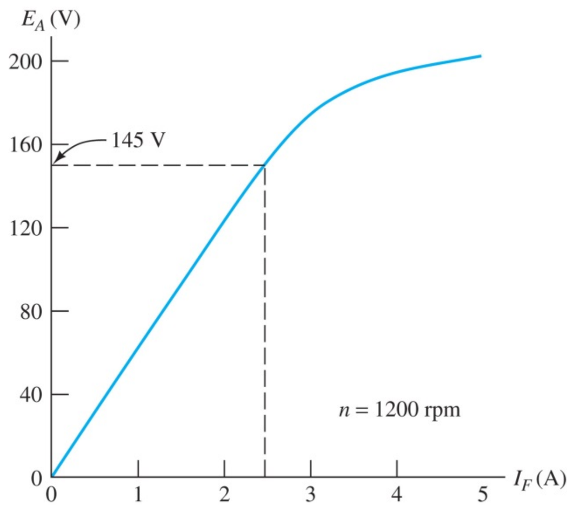


$$\text{fartregulering} = \frac{n_{ingen-last} - n_{full-last}}{n_{full-last}} \cdot 100\% \qquad (1.39)$$

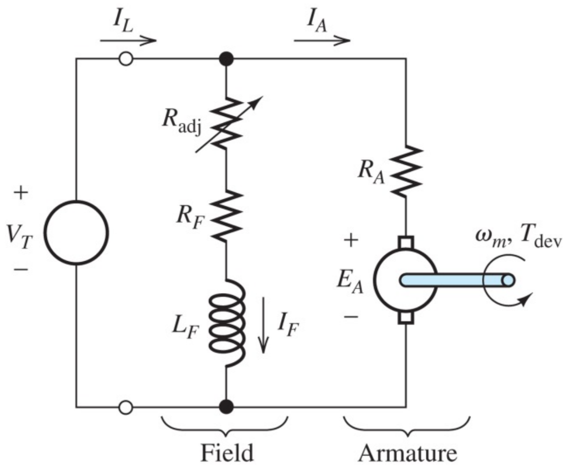
$$\omega_s = \frac{2\omega}{P} \qquad n_s = \frac{120 f}{P} \qquad (1.40)$$

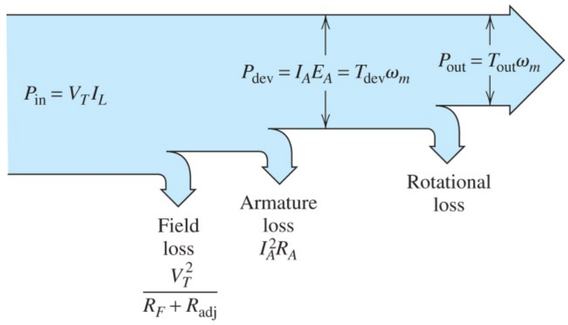


$$\begin{aligned} V_F &= R_F I_F \\ E_A &= K\Phi\omega_m \\ T_{utviklet} &= K\Phi I_A \\ P_{utviklet} &= \omega_m T_{utviklet} \quad , \quad P_{utviklet} = E_A I_A \end{aligned} \qquad (1.41)$$

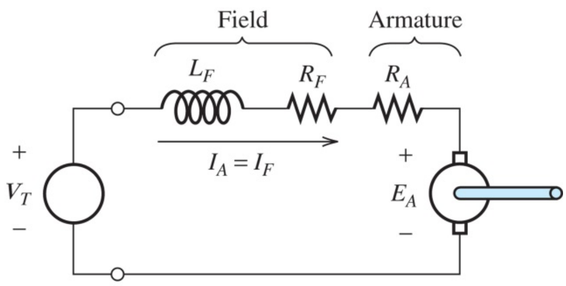


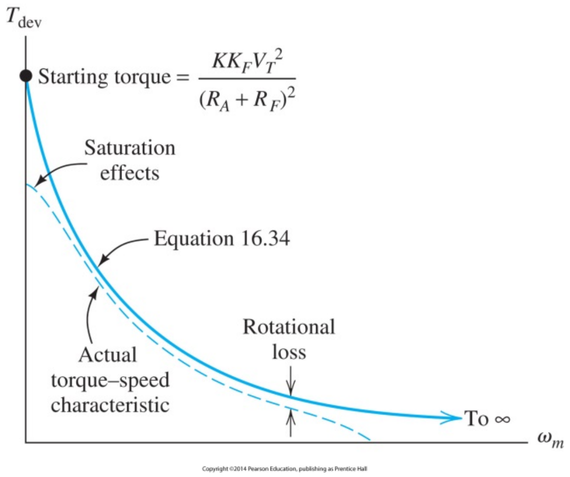
$$\frac{E_{A1}}{E_{A2}} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} \qquad (1.42)$$

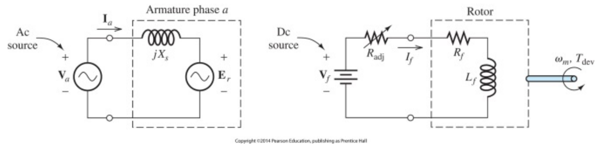




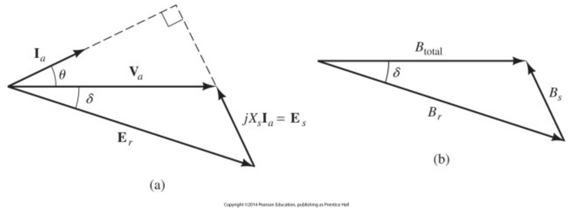
$$T_{utv} = \frac{K\Phi}{R_A}\left(V_T - K\Phi\omega_m\right) \qquad (1.43)$$

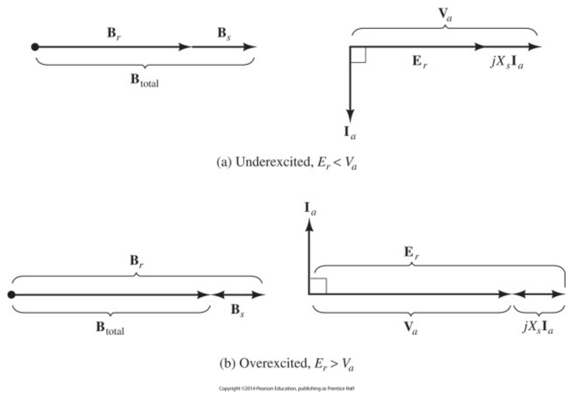




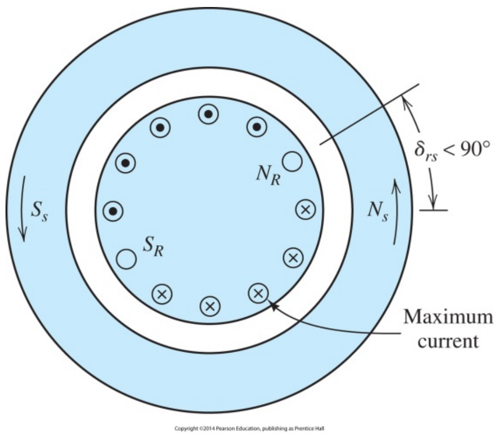


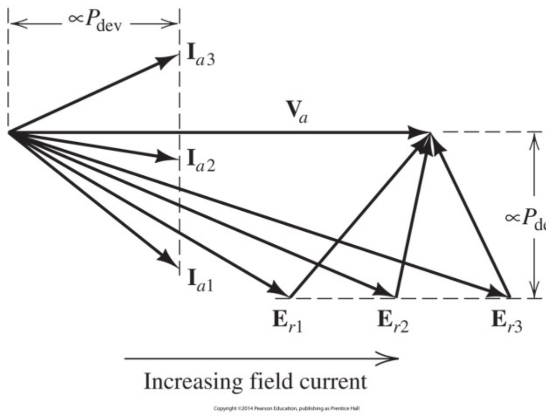
$$T_{utv} = KB_r B_{total} \sin(\delta) \qquad (1.46)$$



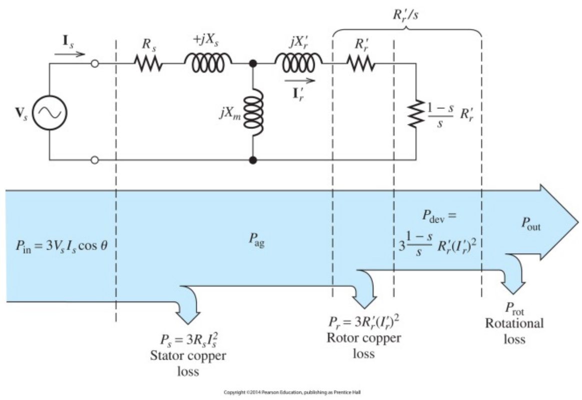


$$T_{utv} = \frac{KK_F V_T^2}{\left(R_A + R_F + KK_F\omega_m\right)^2} \qquad (1.44)$$





$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{n_s - n_m}{n_s} \quad , \quad \omega_{slip} = s\omega \qquad (1.45)$$



Enfase induksjonsmotor: