LØSNINGSFORSLAG
Obliger V-2018

Oblig 3f

a) $v_{Sw} = \frac{q_t}{\phi A}\left(\frac{df_w}{dS_w}\right)_{S_w}$ Buckley-Leverett ligningen

1. v_{Sw} = hastigheten til et plan med konstant vannmetning, S_w

q_t = total flommerate

ϕ = porositet

A = tverrsnittsareal

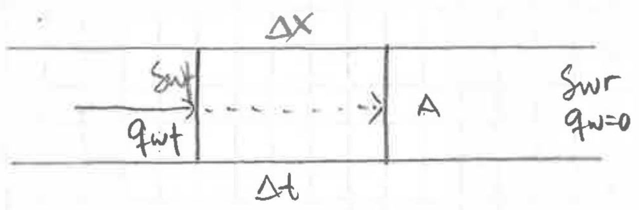
f_w = fraksjonsstrøm av vann

S_w = vannmetning

$\left(\frac{df_w}{dS_w}\right)_{S_w}$ = stigningstallet til fraksjonsstrømskurven i punktet (S_w, f_w)

2. Utlede $\left(\frac{df_w}{dS_w}\right)_{S_{wf}}$

Skisse: Massebalanse i sjokkfronten



1. $q_{wf} = f_{wf} \cdot q_t$

2. $q_{wf} \cdot \Delta t = \Delta X \cdot A \cdot \Phi \cdot (S_{wf} - S_{wr})$

Mengde vann inn i volumelementet $\Phi \cdot A \cdot \Delta x$ i tid Δt:

Setter 1. inn i 2.

$$f_{wf} \cdot q_t \cdot \Delta t = (S_{wf} - S_{wr}) \cdot \Delta x \cdot \Phi \cdot A$$

$$v_{S_{wf}} = \frac{\Delta x}{\Delta t} = \frac{f_{wf} \cdot q_t}{(S_{wf} - S_{wr}) \cdot \Phi \cdot A}$$ fra massebalanse

$$v_{S_{wf}} = \frac{q_t}{\phi A}\left(\frac{df_w}{dS_w}\right)_{S_{wf}}$$ fra B-L

Uttrykkene må være like:

$$\frac{\cancel{q_t}}{\cancel{\phi A}}\left(\frac{df_w}{dS_w}\right)_{S_{wf}} = \frac{f_{wf} \cdot \cancel{q_t}}{(S_{wf} - S_{wr})\cancel{\phi A}}$$

$$\left(\frac{df_w}{dS_w}\right)_{S_{wf}} = \frac{f_{wf}}{(S_{wf} - S_{wr})}$$ s.s.v.

Dette er en betingelse sjokkfronten må oppfylle i følge B-L.

b) 1. Utled f_w for et horisontalt reservoar:

$$f_w = \frac{q_w}{q_w + q_o}$$

$$q_w = -\frac{k_w}{\mu_w} \cdot A \frac{dP}{dL} = -\frac{k_{rw} \cdot k}{\mu_w} A \frac{dP}{dL}$$

$$q_o = -\frac{k_o}{\mu_o} \cdot A \frac{dP}{dL} = -\frac{k_{ro} \cdot k}{\mu_o} A \cdot \frac{dP}{dL}$$

$$\underline{\underline{f_w}} = \frac{q_w}{q_w + q_o} = \frac{-\frac{k_{rw} \cdot k}{\mu_w} A \frac{dP}{dL}}{-\frac{k_{ro} \cdot k}{\mu_o} \cdot A \cdot \frac{dP}{dL} - \frac{k_{rw} \cdot k}{\mu_w} A \frac{dP}{dL}} = \underline{\underline{\frac{1}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}}}}$$

2. Fraksjonsstrømskurven til vann er en funksjon av vannmetning.
Viskositetene til olje og vann er gitt.
Relative permeabiliteter til olje og vann er gitt som funksjon av vannmetning i tabell.

For hver vannmetning beregnes f_w ved å sette inn k_{rw}, k_{ro}, μ_w og μ_o i formelen i 1.

Deretter plottes f_w som funksjon av S_w

3. a) Tid til vanngjennombrudd, t_{BT} :

Fra tangent og fraksjonsstrømskurve finner vi:

$S_{wf} = 0.64$

$f_{wf} = 0.86$

$$t_{BT} = \frac{L}{V_{swf}} = \frac{L}{\frac{q_t}{\phi A}\left(\frac{df_w}{dS_w}\right)_{swf}} = \frac{L}{\frac{Q_w \cdot B_w}{\phi \cdot A}\left(\frac{f_{wf}}{S_{wf} - S_{wr}}\right)}$$

$$= \frac{1000\ m}{\frac{200\ Sm^3/d \cdot 1.0\ m^3/Sm^3}{0.26 \cdot 10\,000\ m^2} \cdot \frac{0.86}{0.64 - 0.16}} = 7255{,}8 = \underline{\underline{7256\ dager}}$$

$\approx 19{,}9$ år

b) Produsert olje:

$$\underline{\underline{N_p}} = \frac{q_o \cdot t_{BT}}{B_o} = \frac{Q_w \cdot B_w \cdot t_{BT}}{B_o} = \frac{200\ Sm^3/d \cdot 1.0\ m^3/Sm^3 \cdot 7255.8\ d}{1.5\ m^3/Sm^3}$$

$$= \underline{\underline{967\,440\ Sm^3}}$$

c) Produserbar olje: $\frac{\phi A L(1 - S_{wr} - S_{or})}{B_o}$

$$= \frac{0.26 \cdot 10\,000\ m^2 \cdot 1000\ m \cdot (1 - 0.16 - 0.21)}{1.5\ m^3/Sm^3}$$

$$= \underline{\underline{1\,092\,000\ Sm^3}}$$

Utvinnings %: $\frac{N_p}{\text{Produserbar olje}} \cdot 100\,\%$

$$= \frac{967\,440\ Sm^3}{1\,092\,000\ Sm^3} \cdot 100\,\% = \underline{\underline{88.6\,\%}}$$

d) Vannkutt, WOR (Sm^3/Sm^3)

$$\underline{\underline{WOR}} = \frac{Q_w}{Q_o} = \frac{\frac{q_w}{B_w}}{\frac{q_o}{B_o}} = \frac{q_w \cdot B_o}{q_o \cdot B_w} = \frac{q_t \cdot f_{wp} \cdot B_o}{q_t \cdot (1-f_{wp}) \cdot B_w}$$

$$= \frac{0.86 \cdot 1.5\,m^3/Sm^3}{(1-0.86) \cdot 1.0\,m^3/Sm^3} = \underline{\underline{9.21\ \frac{Sm^3}{Sm^3}}}$$

c) Ved WOR = 30 Sm^3/Sm^3 skal produksjonen avsluttes

1. Produksjonstid, t (år)

$$WOR = \frac{f_{wp} \cdot B_o}{(1-f_{wp}) \cdot B_w}$$

$$WOR \cdot (1-f_{wp}) = \frac{f_{wp} \cdot B_o}{B_w}$$

$$\frac{1-f_{wp}}{f_{wp}} = \frac{B_o}{B_w \cdot WOR}$$

$$\frac{1}{f_{wp}} - 1 = \frac{B_o}{B_w \cdot WOR}$$

$$\frac{1}{f_{wp}} = \frac{B_o + B_w \cdot WOR}{B_w \cdot WOR}$$

$$f_{wp} = \frac{B_w \cdot WOR}{B_o + B_w \cdot WOR} = \frac{1.0\,m^3/Sm^3 \cdot 30\,Sm^3/Sm^3}{1.50\,m^3/Sm^3 + 1.0\,m^3/Sm^3 \cdot 30\,Sm^3/Sm^3}$$

$\underline{f_{wp} = 0.952}$ $\qquad S_{wp} = 0.71$ fra kurve

Trekker så en tangent til kurven i $f_{wp} = 0.952$.
Stigningstallet til tangenten er $\frac{0.08}{0.1} = 0.8$

$$t = \frac{L}{v_{Swp}} = \frac{L}{\frac{q_t}{\phi A}\left(\frac{df_w}{dS_w}\right)_{Swp}} = \frac{1000\,m}{\frac{Q_w \cdot B_w}{\phi A} \cdot 0.8}$$

$$t = \frac{1000\,m}{\frac{200\,Sm^3/d \cdot 1.0\,m^3/Sm^3}{0.26 \cdot 10000\,m^2} \cdot 0.8} = 16250\ d = \underline{\underline{44.5\ år}}$$

2. Produsert olje (Sm^3):

$$\underline{\underline{N_p}} = \frac{\Phi A L (\bar{S_w} - S_{wr})}{B_o} = \frac{0.26 \cdot 10000\,m^2 \cdot 1000\,m \cdot (0.77 - 0.16)}{1.50\ m^3/Sm^3}$$

$$= \underline{\underline{1\,057\,333.3\ Sm^3}}$$

3. % utvinning av produserbar olje:

$$\frac{\frac{\cancel{\Phi A L}(\bar{S_w} - S_{wr})}{\cancel{B_o}}}{\frac{\cancel{\Phi A L}(1 - S_{wr} - S_{or})}{\cancel{B_o}}} \cdot 100\%$$

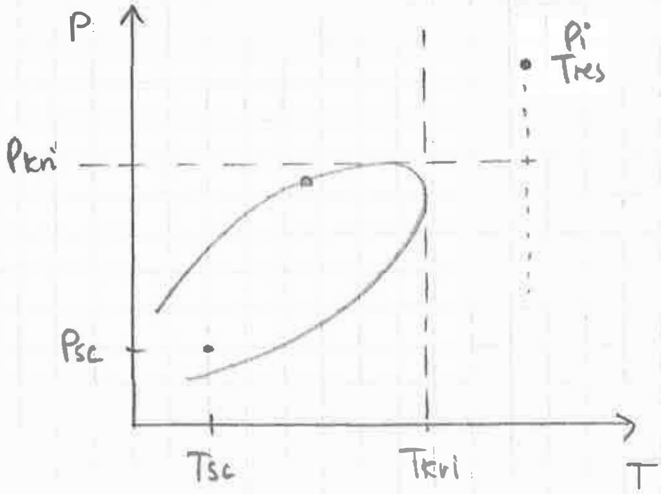
$$\frac{(\bar{S_w} - S_{wr})}{(1 - S_{wr} - S_{or})} \cdot 100\% = \frac{0.77 - 0.16}{1 - 0.16 - 0.21} \cdot 100\% = \underline{\underline{96.8\ \%}}$$

% utvinning av IOIP:

$$\frac{(\bar{S_w} - S_{wr})}{(1 - S_{wr})} \cdot 100\% = \frac{0.77 - 0.16}{1 - 0.16} \cdot 100\% = \underline{\underline{72.6\ \%}}$$

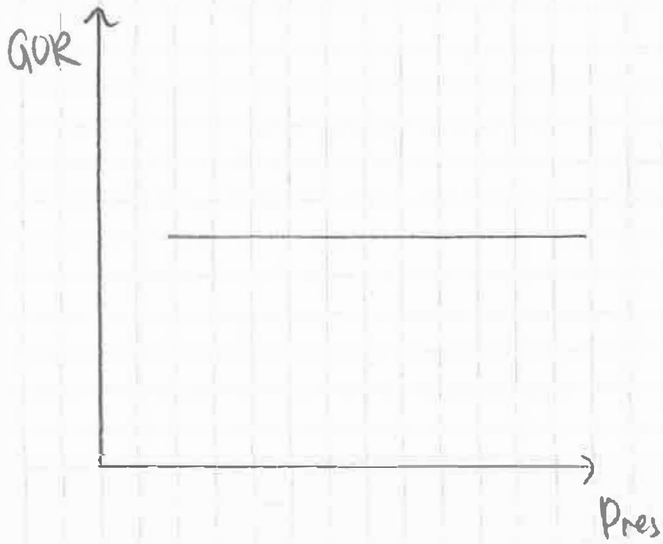
Oblig 2

a) 1. Våt gass reservoar :



- $T_{res} > T_{kri}$
- P_{sc}, T_{sc} er inni tofasekonvolutten

2. $GOR = f(P_{res})$



GOR er konstant fordi reservoarfluid med konstant komposisjon strommer inn i bronn og separerer i samme forhold gass og olje ved standardbetingelser

b) Bestem IGIP og IOIP pr 10 000 m³ brutto reservoarvolum

$$\underline{HCPV} = V_b \cdot \Phi \cdot (1 - S_{wr}) = 10000 \cdot 0.25 \cdot (1 - 0.10)$$
$$= \underline{2250 \text{ m}^3}$$

Initielt antall mol reservoarfluid:

$$PV = ZnRT$$

$$n_i = \frac{P_i \cdot HCPV}{Z_i \cdot R \cdot T_{res}} = \frac{50\,000\ kPa \cdot 2250\ m^3}{1.236 \cdot 8{,}3145\ \frac{kPa \cdot m^3}{kgmol \cdot °K} \cdot (100 + 273{,}15)\ °K}$$

$$= \underline{29\,336{,}9\ kgmol}$$

Vi må finne molfraksjonene V og L for å vite hvor stor del av n_i som går til olje og gass.

Tar utgangspunkt i GOR.

$$GOR_{tot} = GOR_{sep} + GOR_{tank}$$
$$= 6500\ Sm^3/Sm^3 + 500\ Sm^3/Sm^3 = \underline{7000\ Sm^3/Sm^3}$$

Antall mol i 1 Sm^3 STO:

$$n_{STO} = \frac{m_{STO}}{M_{STO}} = \frac{\rho_{STO} V_{STO}}{M_{STO}} = \frac{750\ kg/m^3 \cdot 1\ Sm^3}{105} = \underline{7.143\ kgmol}$$

Per 1 Sm^3 STO, produseres 7000 Sm^3 gass

Antall mol gass produsert:

$$V_{g,sc} = n_g \cdot V_m \Rightarrow n_g = \frac{V_{g,sc}}{V_m} = \frac{7000\ Sm^3}{23.6447\ Sm^3/kgmol}$$

$$\underline{n_g = 296{,}049\ kg\ mol}$$

Totalt antall mol produsert:

$$n_{tot} = n_{STO} + n_g = 7.143 + 296{,}049 = \underline{303{,}192\ kgmol}$$

Molfraksjon STO : $L = \frac{n_{STO}}{n_{tot}} = \frac{7,143}{303.192} = 0.0236$

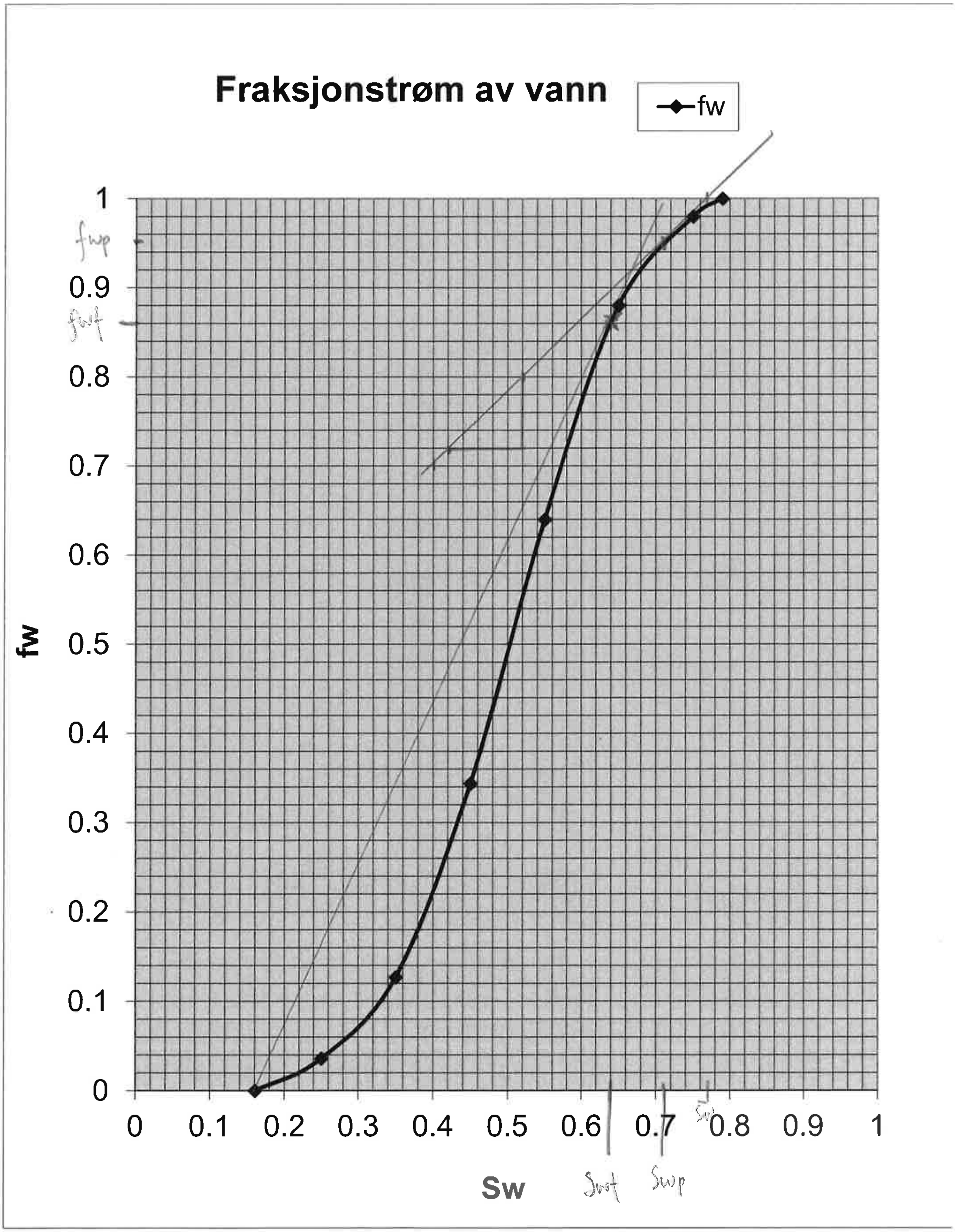
Molfraksjon gass : $V = \frac{n_g}{n_{tot}} = \frac{296,049}{303.192} = 0.9764$

$\underline{IGIP} = n_i \cdot V \cdot V_m = 29\,336.9 \text{ kgmol} \cdot 0{,}9764 \cdot 23{,}6447 \, \text{Sm}^3/\text{kgmol}$

$= \underline{\underline{677\,291{,}8 \text{ Sm}^3}}$

$GOR_i = \frac{IGIP}{IOIP}$

$\Rightarrow \underline{\underline{IOIP}} = \frac{IGIP}{GOR} = \frac{677\,291{,}8 \text{ Sm}^3}{7000 \text{ Sm}^3/\text{Sm}^3} = \underline{96{,}76 \text{ Sm}^3}$



Fraksjonstrøm av vann
fw
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Sw
fwp
fwf
Swf
Swp

Oblig 1 + 3

a)

$$q = -\frac{k\,A}{\mu}\left[\frac{dp}{dx} + \frac{1}{G}\rho\,g\,\frac{dz}{dx}\right], \tag{1}$$

where A is the cross section, k is the permeability, dp the pressure drop, q is the volumetric flow rate, μ the viscosity and dx is the length. $g = 980$ cm^2/s, ρ is the density of the fluid, z is the vertical distance from the datum plane. The units are: cm (length), s (time), cP (viscosity), Darcy (permeability), atm (pressure), gram/cm^3 (density), and $G = 1.0133 \cdot 10^6$.

b)

$$\begin{aligned} q\frac{159000}{86400} &= -\frac{(30.48)^2\,A\,10^{-3}k}{\mu}\frac{(14.696)^{-1}dp}{(30.48)dx} \\ q &= -\,0.001127\,\frac{A\,k}{\mu}\frac{dp}{dx} \end{aligned} \tag{2}$$

$$1\text{D} = \frac{1\text{cm}^3/\text{s}\,1\text{cp}}{1\text{cm}^2\,1\text{atm/cm}} = \frac{\text{cm}^2\,\text{cp}}{\text{s atm}} = \frac{10^{-12}}{1.01325}\text{m}^2 = 0.987\mu\text{m}^2. \tag{3}$$

c)
1. From $q\cdot\rho$=constant and the ideal gas law $p\,V$=constant, we get $q\,p$ = constant $= q_b\,p_b$, (where the subscript b indicates a reference state, it could be the inlet or the outlet). Darcy's law: $q = -k\,A/\mu\,(d\,p/d\,x)$. q_b og p_b henviser til en valgt referansetilstand. Integrating this equation, we get the result given in the problem.
2. The total volumetric flow rate has to be equal to the sum of the volumetric flow rate in each layer:

$$\begin{aligned} q &= q_1 + q_2 + q_3 \\ &\frac{\overline{k}\,(h_1+h_2+h_3)\,B}{\mu_g}\frac{(p_1^2-p_2^2)}{p_b\,L} \\ &= \frac{k_1\,h_1\,B}{\mu_g}\frac{(p_1^2-p_2^2)}{p_b\,L} + \frac{k_2\,h_2\,B}{\mu_g}\frac{(p_1^2-p_2^2)}{p_b\,L} + \frac{k_3\,h_3\,B}{\mu_g}\frac{(p_1^2-p_2^2)}{p_b\,L}, \\ \overline{k} &= \frac{1}{h}(k_1\,h_1 + k_2\,h_2 + k_3\,h_3)\,. \end{aligned} \tag{4}$$

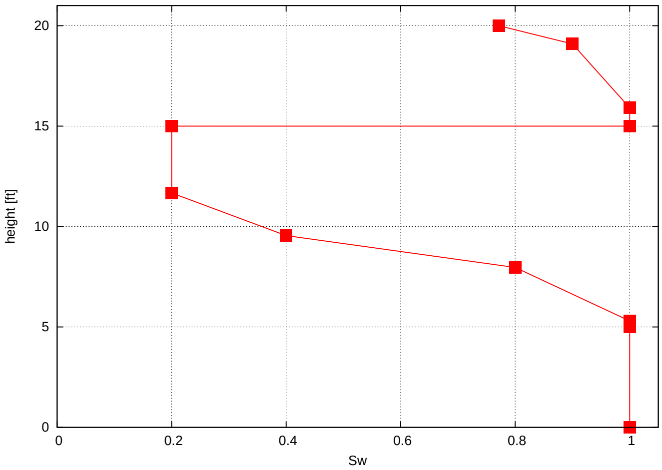
d) Effective permeability is $\overline{k} = (200\cdot 5 + 1000\cdot 10 + 200\cdot 15)$mD/30=466.67mD, and the conversion factor:

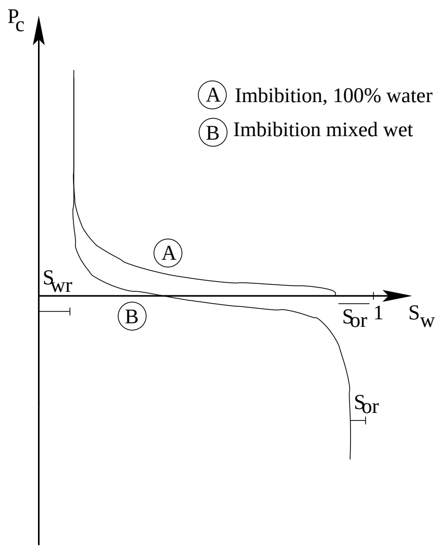
$$\begin{aligned} q_b\frac{(30.48)^3}{24\cdot 60\cdot 60} &= \frac{\overline{k}\,A(30.48)^2}{\mu_g}\frac{1}{2\,p_b(0.068046)\,L(30.48)}\left(p_1^2-p_2^2\right)(0.068046)^2, \\ q_b &= 3.16414\frac{\overline{k}\,A}{\mu_g}\frac{1}{p_b\,L}\left(p_1^2-p_2^2\right), \end{aligned} \tag{5}$$

Using the numbers given in the text we find: $q_b = 7.5945\cdot 10^6$ ft^3/day.

e)
1. $q_2/q_1 = k_2A_2/(k_1A_1)$, $q_3/q_1 = k_3A_3/(k_1A_1)$. The ratio is the same for gas flow.
2. Turbulence: Darcy law is only valid for laminar flow and that momentum (inertial forces) can be neglected (low Reynolds number). At high Reynold numbers turbulence may occour, and the turbulent flow give rise to additional loss of energy and pressure drop not accounted for in Darcy's law.
3. Slip at the pore wall: At low pressure the mean free-path of the gas molecules may be of the same size as the pore size. This leads to a non zero fluid velocity at the pore wall, which is assumed in Darcy law. Fluid speed will therefore be higher than expected from Darcy law, and can be corrected by introducing a Klinkenberg term.

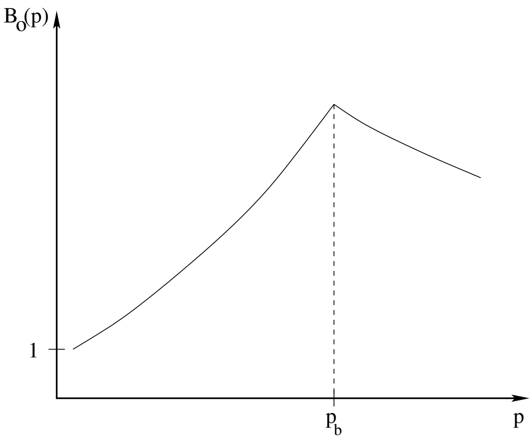
f) At the free water level (FWL) the oil pressure is equal to the water pressure $p_o = p_w = p'$, hence $p_c = 0$. However, the oil oil needs to exceed an entry pressure to displace water from the pores due to surface forces, therefore the oil-water contact is located above (for a water-wet system) than the FWL. Above the FWL the capillary pressure is given by: $p_c(S_w) = (\rho_w - \rho_o)gh$.





g)

h) $B_o = \Delta V^S_{o,o} / \Delta V^R_o$



i) From the given formulas: (Note that $R_p = R_s$)

$$N = \frac{N_p B_o}{E_o + E_c} \tag{6}$$

Using the data given, we find:

$$\begin{aligned}
E_c &= B_{oi}\frac{c_w S_w + c_p}{1 - S_w}\Delta p = B_{oi}\frac{8.6 \cdot 10^{-6} S_w + 3.3 \cdot 10^{-6}}{1 - 0.43}(7150 - 4500) \\
&= 0.032595 && (7) \\
E_o &= B_o - B_{oi} = 1.850 - 1.743 = 0.107 && (8) \\
N &= \frac{1.85043.473 \text{ MMSTB}}{0.107 + 0.0326} = 576 \text{ MMSTB} && (9)
\end{aligned}$$

j) First we need to find the volume of oil at reservoir conditions: V_o^R=576·1.7413MM bbl $\simeq$ 1003·10^6 bbl. The total pore volume of fluid is then : $V_p = V_o/S_o = V_o/(1-S_w) = 1760 \cdot 10^6$ bbl= 280·10^6m^3. The average height is then 280/20$\simeq$ 14 m.