ELE320 - Aero

Preparatory Assignment 1

Topics Covered

- Introduction to Quanser Aero
- Low pass signal filtering
- Accelerometer and Gyroscopes
- Approximating position
- Obtaining the equations of motion of the Aero thruster
- Creating and validating a sub-system model
- Model validation

0. Introduction

The Quanser Aero is a dual-rotor aerospace development application with reconfigurable dynamic components for teaching control concepts at an undergraduate level.

Due to its reconfigurable base, the Quanser Aero can be used to develop control systems for various aerospace applications. The available configurations are: 1 degrees of freedom (DOF) attitude control, 2 DOF half-quad, and 2 DOF helicopter. The equipment consists of two propellers, powered by DC motors. Each motor is equipped with high-resolution optical encoders for velocity measurements. The Aero base has encoders on each rotational joints for measuring the pitch and yaw positions. In a realistic aerospace application, it is not feasible to measure the pitch and yaw positions using encoders. For that reason, the Aero is equipped with an inertial measurement unit (IMU) which can be used to percept the pitch and yaw motion in a scenario closer to reality. In such scenarios, the encoders are mainly used as a reference to verify the IMU data. However, when learning fundamental control concepts, the encoders are convenient to use as feedback sensors. To monitor the power consumption of the motors, the available current and voltage sensors can be utilized.

In ELE320 you will experiment with the 1 DOF configuration to enhance your skills regarding modeling, system identification, and PID control. The University of Stavanger possess two units of the physical Quanser Aero, as well as access to a virtual model of the system. In this course, the mandatory assignments will be conducted on the virtual model. Once you have a working model on the virtual Aero, you can implement it on the physical plant (not mandatory).

1. Measurement and Filtering

1.1 Inertial Measurement Unit

An Inertial Measurement Unit (IMU) can be a combination of accelerometer, gyroscope, and magnetometer. In this case, the IMU is a combination of an accelerometer and a gyroscope. Measuring the acceleration in each linear axis along with the angular velocity about those axes allows for a complete description of the attitude of the sensor.

The internal IMU in the Quanser Aero represents how a flight system would sense its attitude more realistically than the encoders since a vehicle in flight lacks an unmoving base from which such encoder measurements could be made.

The Quanser Aero IMU is located in the center of the Aero body, in line with the axes of rotation. For the purposes of measuring and controlling pitch, we are primarily interested in the acceleration along the X and Z axes, and the rotation about the Y axis as shown in Figure 1.1.

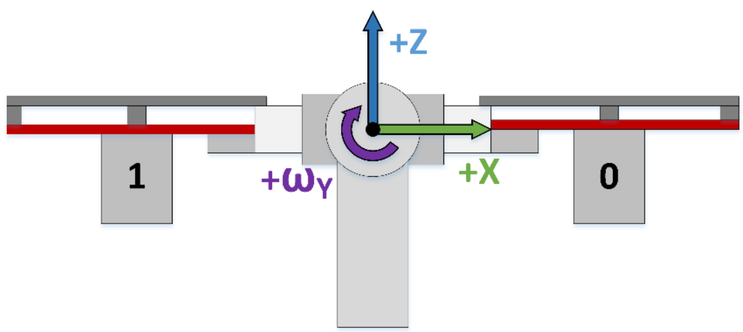

Figure 1.1: Pertinent IMU axes

1.2 Measuring Pitch Using Gravity

Since the aero body is immobile, it can be assumed that the only linear acceleration affecting the IMU will be a result of gravity. The accelerometer measures the acceleration along the three principle axes, however since the Y axis will always be perpendicular to the acceleration of gravity we are only interested in the X and Z axis accelerations. If the Aero body is pitched to an arbitrary angle θ the acceleration of gravity a_g can be separated into two perpendicular accelerations acting along the X and Z axes according to the equations

$$a_x = a_g \sin(\theta) \tag{1.1}$$

$$a_z = a_g \cos(\theta) \tag{1.2}$$

$$\theta = \tan^{-1}\left(\frac{a_x}{a_z}\right) \tag{1.3}$$

1.3 Approximating Pitch Using Angular Velocity

In many cases, such as when a vehicle is accelerating vertically, or when measuring yaw, there is no way to use acceleration to calculate the angular position of the vehicle. In this case the position is often approximated by integrating the angular velocity measured by the gyroscope. In this way the angle at time t is given by

$$\theta_t = \theta_0 + \int_0^t \omega dt \tag{1.4}$$

1.4 Low-pass filtering

To accurately calculate the attitude of a vehicle from sensor readings, noise must be eliminated as much as possible. In the case of the Quanser Aero most of the noise on the IMU signals is due to vibration caused by the propellers. Since the frequency of the vibration is significantly greater than that of the attitude signals the noise can be attenuated using a low-pass filter. A first-order low-pass filter has the form

$$G(s) = \frac{\omega_f}{s + \omega_f} \tag{1.5}$$

Where ω_f is the cut-off frequency of the filter in radians per second (rad/s). A first-order filter of this form attenuates signals at the cut-off frequency by $-3\ dB\ \approx 30\%$. Signals at frequencies higher than ω_f are attenuated by approximately $-20\ dB/decade$, meaning that a signal at a frequency of $10\omega_f$ will be attenuated by about $-20\ dB \approx 90\%$.

2. First Principle Modeling

The Quanser Aero has two thruster modules, each of which is driven by a DC motor. The motor armature circuit schematic for one of the thrusters is shown in Figure 2.1 and the electrical and mechanical parameters are given in Table 2.1. The DC motor shaft is connected to the propeller hub. The hub is a collet clam used to mount the propeller to the motor and has a moment of inertia of J_h. A propeller is attached to the output shaft with a moment of inertia of J_p.

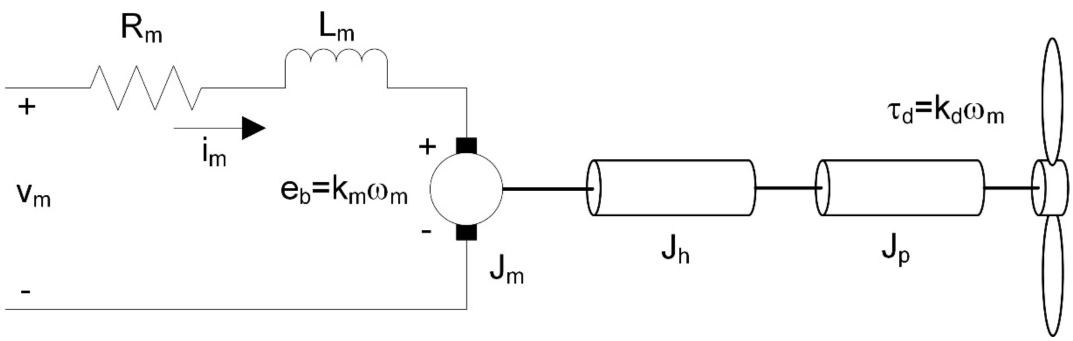

Figure 2.1: Quanser Aero thruster motor and load

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \tag{2.1}$$

The torque exerted by drag and air resistance has been simplified into an experimentally derived coefficient of the speed of the propeller. In this model the drag torque T_d which opposes the motor torque is given by:

$$\tau_d(t) = k_d \omega_m(t) \tag{2.2}$$

Symbol	Description	Value
DC Motor		
R_m	Terminal resistance	$8.4\ \Omega$
k_t	Torque constant	0.053 N · m/A
k_m	Motor back-emf constant	0.053 V/(rad/s)
J_m	Rotor inertia	2.0×10^{-6} kg · m^2
L_m	Rotor inductance	1.16 mH
Propeller		
k_d	Drag/Air resistance coefficient	2×10^{-5} N · m/(rad/s)
J_h	Propeller hub inertia	5.04×10^{-8} kg · m^2
J_p	Propeller inertia	5.6×10^{-5} kg · m^2

Table 2.1: Quanser Aero system parameter

Using Kirchoff's Voltage Law, we can write the following equation:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0 \tag{2.3}$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0 \tag{2.4}$$

Solving for $i_m(t)$, the motor current can be found as:

$$i_m(t) = \frac{v_m(t) - k_m \omega_m(t)}{R_m} \tag{2.5}$$

In a DC motor, the torque delivered by the motor is proportional to the current through the windings. Based on the applied current, the torque τ_m is:

$$\tau_m(t) = k_m i_m(t) \tag{2.6}$$

Applying Newton's second law for rotation, the motor shaft equation is expressed as:

$$J_{eq}\ddot{\theta} = \sum_i \tau_i \tag{2.7}$$

$$J_{eq}\dot{\omega}_m(t) = \tau_m(t) - \tau_d(t) \tag{2.8}$$

Where J_{eq} is total moment of inertia acting on the motor shaft.

$$J_{eq} = J_h + J_p + J_m \tag{2.9}$$

3. System Identification

Unlike a DC motor, the Quanser Aero has to be characterized with at least a second-order model. The equation of motion is derived from first principles and then used to obtain the transfer function representing the thrust to position dynamics of the Quanser Aero.

3.1 Torques Acting on the Quanser Aero

The free-body diagram of a Quanser Aero in the 1-DOF configuration that pivots about the pitch axis is shown in Figure 3.1.

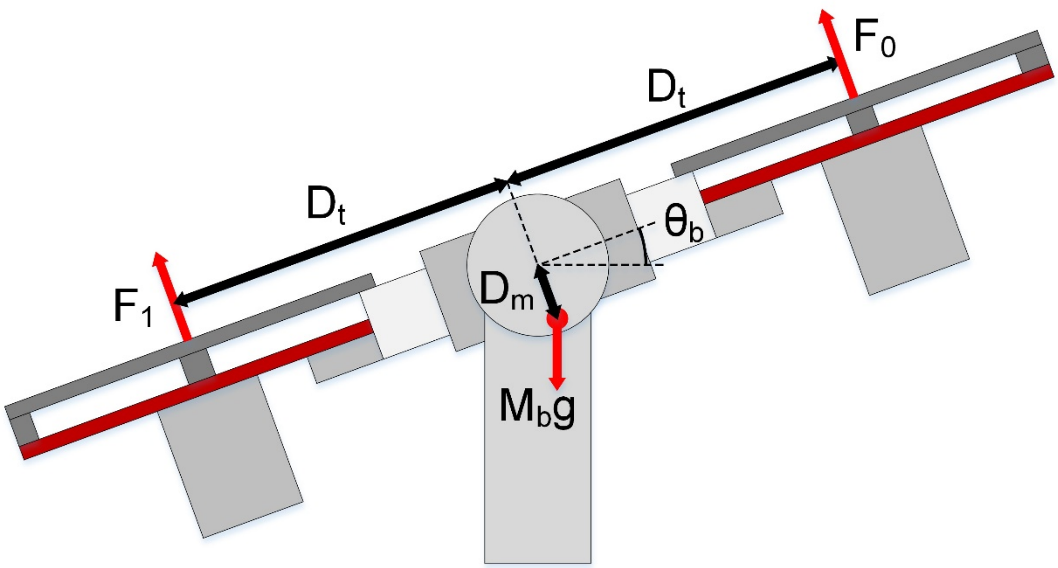

Figure 3.1: Free-body diagram of 1-DOF Quanser Aero

Symbol	Description	Value
m_b	Aero body mass	1.15 kg
D_t	Thrust displacement	0.158 m
D_m	Center of mass displacement	0.0071 m

Table 3.1: Quanser Aero system parameters

The torques acting on the rigid body system at rest can be described by the equation

$$\tau_t - M_b g(D_m \sin(\theta_b)) \tag{3.1}$$

where D_m is the distance below the plane of the Aero body of the center of mass as depicted in Figure 3.1. The thrust torque is given by

$$\tau_t = F_0 D_t - F_1 D_t \tag{3.2}$$

where F_0 and F_1 are the thrust forces perpendicular to the aero body generated by thrusters 0 and 1 respectively.

Thus the complete torque equation becomes

$$F_0 D_t - F_1 D_t - M_b g(D_m \sin(\theta_b)) = 0 \tag{3.3}$$

The forces exerted by the thrusters are related to the angular velocity by the formula

$$F = K_t \omega \tag{3.4}$$

where K_t is the thrust constant of the thruster. Since the motors act in opposition to each other along the same degree of freedom, the motor speed ω_0 and ω_1 can be replaced with a single control variable:

$$\omega_m = \omega_0 - \omega_1 \tag{3.5}$$

Thus with respect to the differences applied to the motors, the torque equation becomes

$$K_t \omega_m D_t - M_b g(D_m \sin(\theta_b)) = 0 \tag{3.6}$$

Original work by Quanser – modified by Didrik Efjestad Fjereide

© 2016 Quanser Inc., All rights reserved.

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com

This document and the software described in it are provided subject to a license agreement. Neither the software nor this document may be used or copied except as specified under the terms of that license agreement. Quanser Inc. grants the following rights: a) The right to reproduce the work, to incorporate the work into one or more collections, and to reproduce the work as incorporated in the collections, b) to create and reproduce adaptations provided reasonable steps are taken to clearly identify the changes that were made to the original work, c) to distribute and publically perform the work including as incorporated in collections, and d) to distribute and publicly perform adaptations. The above rights may be exercised in all media and formats whether now known or hereafter devised. These rights are granted subject to and limited by the following restrictions: a) You may not exercise any of the rights granted to You in above in any manner that is primarily intended for or directed toward commercial advantage or private monetary compensation, and b) You must keep intact all copyright notices for the Work and provide the name Quanser Inc. for attribution. These restrictions may not be waved without express prior written permission of Quanser Inc.