ELE320 - Aero

Preparatory Assignment 2

Topics Covered

- Aero pitch control.
- Proportional-integral-derivative (PID) compensator.
- Manual PID tuning.
- Second-order system approximation.
- Controlling second-order systems.
- PID design by specification.

Prerequisites

- First Principles Modeling laboratory assignment.
- System Identification laboratory assignment.

4. Qualitative PID Control

4.1 Proportional Action

A proportional action drives the plant based on the difference between the current output of the system and the desired outut. This difference is amplified by the proportional gain K_p which can be tuned either experimentally or calculated based on system requirements such as rise time. Bigger proportional gain will result in a system with a shorter rise time, that is, the time needed for the system to reach the desired output. However, since the system is constantly accelerating towards the set point, large proportional gains will generally lead to a system with large overshoot, and which oscillates for a long time.

4.2 Derivative Action

To deal with the overshoot and oscillation caused by a proportional action, many systems implement a derivative action in parallel. This action drives the plant based on the rate of change of the error signal. As with proportional control, this derivative is amplified by the derivative gain K_d. The derivative action effectively acts as added damping in underdamped systems. To improve the stability of systems with derivative action, filtering is often added to prevent spikes in the derivative due to signal noise.

4.3 Integral Action

In many cases, the combination of proportional and derivative gain will result in a system which does not settle sufficiently close to the setpoint. In this case, an integral compensator may be added. This compensator drives the system based on the integral of the error over time magnified by the integral gain K_i. This component of the controller increases the longer the system remains far from the setpoint.

4.4 PID Control

The complete proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt} \tag{4.1}$$

The corresponding block diagram is given in Figure 4.1. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller Equation 4.1 can also be described by the transfer function

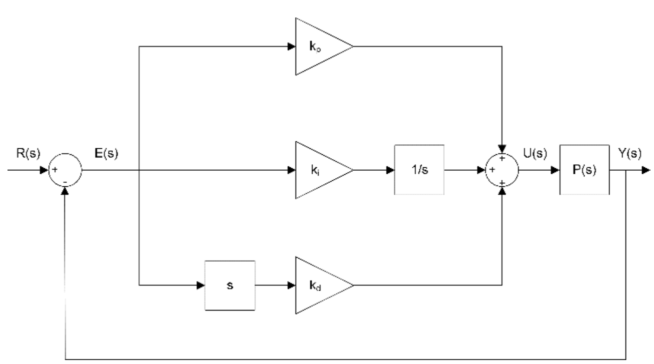

Figure 4.1: Block diagram of PID control

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{4.2}$$

5 PID Control to Specification

5.1 Second-Order System Model

For the purpose of this part of the lab we will consider the Aero under the closed loop unity feedback control to be a second-order system with a transfer function of the form

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5.1}$$

where ω_n is the natural undamped frequency and ζ is the damping ratio. The properties of its response depend on the values of the ω_n and ζ parameters. Consider when a second-order system is subjected to a unit step in the input signal $u(t)$. The system response is shown in Figure 5.1, where the blue line is the response (output) $y(t)$ and the red line is the step input $u(t)$.

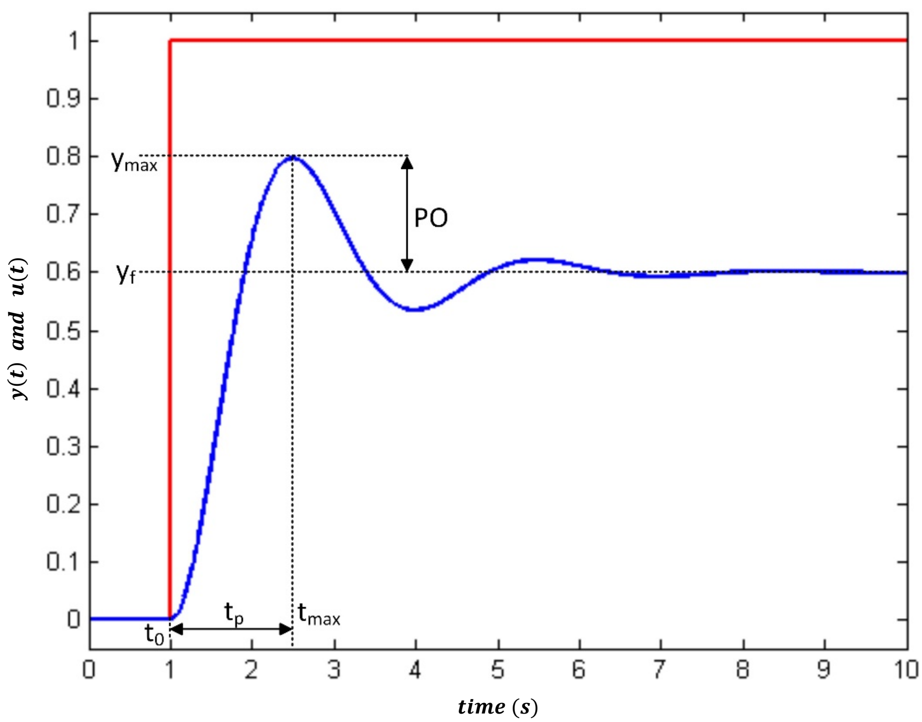

Figure 5.1: Response of a second order system to a unit step

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 5.1, where the system settles at a final value of y_f the percent overshoot is found using Equation 5.2

$$PO = 100\left(\frac{y_{max} - y_f}{y_f}\right) \tag{5.2}$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the Equation 5.3

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \tag{5.3}$$

From the initial step time t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0 \tag{5.4}$$

This is called the peak time of the system and it depends on both the damping ratio and natural frequency of the system. It can be derived analytically as

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{5.5}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

Another useful measure of system response is how long the system takes to settle. This is defined as the time required for the system to settle within a certain percentage δ of y_f. For example, the 5% settling time is computed using $\delta = 0.05$. Since the second order response is bounded by $e^{-\zeta \omega_n T_s}$, this occurs at approximately

$$e^{-\zeta \omega_n T_s} = \delta \tag{5.6}$$

5.2 Controlling a second-order system

Consider a general second order system with the open loop transfer function

$$P(s) = \frac{a}{s^2 + bs + c} \tag{5.7}$$

and a general PID controller described by the transfer function

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{5.8}$$

where K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain. The direct loop transfer function from $E(s)$ to $Y(s)$ is as

$$L(s) = \frac{a\left(k_d s^2 + k_p s + k_i\right)}{s(s^2 + bs + c)} \tag{5.9}$$

and the characteristic polynomial of the closed-loop transfer function is:

$$p(s) = s^3 + (b + ak_d)s^2 + \left(c + ak_p\right)s + ak_i \tag{5.10}$$

This equation can be set equal to a prototypical third-order equation

$$L(s) = (s + p_0)(s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{5.11}$$

where p_0 is the position of the third pole and ω_n and ζ describe the response of a second-order system. If the pole p_0 is chosen to be faster than the two poles associated with the second-order polynomial in Equation (5.11), then by virtue of the dominant pole approximation the closed-loop system would behave as a second order system. On the contrary, if the pole p_0 is chosen to be slower than the second-order polynomial's roots, then the closed-loop system would behave as a first-order system characterized by the time constant $-1/p_0$.

The following equalities can be determined by gathering equal powers of s

$$k_d = \frac{2\zeta\omega_n + p_0 - b}{a} \tag{5.12}$$

$$k_p = \frac{\omega_n^2 + 2\zeta\omega_n p_0 - c}{a} \tag{5.13}$$

$$k_i = \frac{\omega_n^2 p_0}{a} \tag{5.14}$$

which allow computing the gains for the PID controller so that some desired closed-loop behavior is achieved.

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