BYG140 KONSTRUKSJONSMEKANIKK 1
Assignment (1) - Solutions
(Statics Ch 1: Basic Principles , Ch 2: Force Vectors & Ch 3: Equilibrium of a particle)

Question 1

***1–8.** If a car is traveling at 88.5 km/h, determine its speed in meters per second.

$$88.5\ \text{km/h} = \left(\frac{88.5\ \text{km}}{1\ \text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 24.6\ \text{m/s} \qquad \text{Ans}$$

Question 2

1–10. What is the weight in newtons of an object that has a mass of: (a) 10 kg, (b) 0.5 g, and (c) 4.50 Mg? Express the result to three significant figures. Use an appropriate prefix.

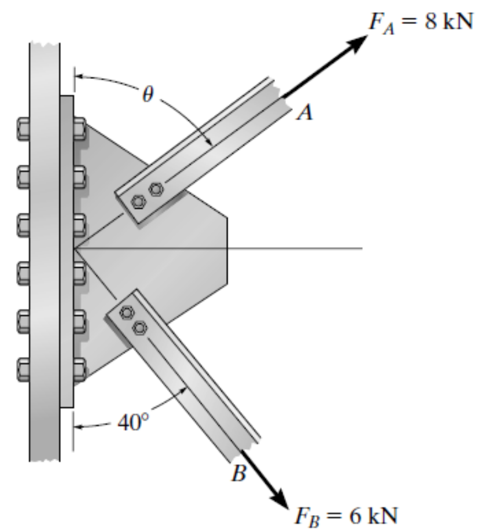
(a) $W = (9.81\ \text{m/s}^2)(10\ \text{kg}) = 98.1\ \text{N}$ **Ans**

(b) $W = (9.81\ \text{m/s}^2)(0.5\ \text{g})(10^{-3}\ \text{kg/g}) = 4.90\ \text{mN}$ **Ans**

(c) $W = (9.81\ \text{m/s}^2)(4.5\ \text{Mg})(10^3\ \text{kg/Mg}) = 44.1\ \text{kN}$ **Ans**

Question 3

•2–9. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

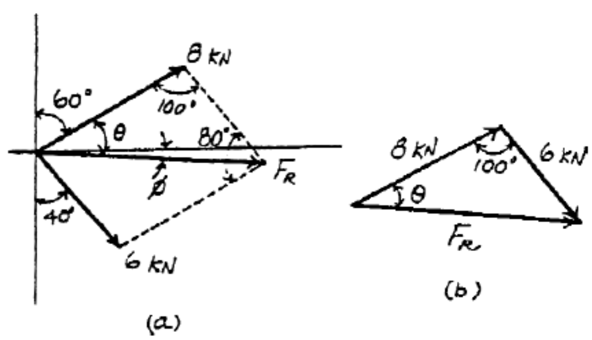
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans}$$

The angle θ can be determined using law of sines [Fig. (b)].

$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16°$$

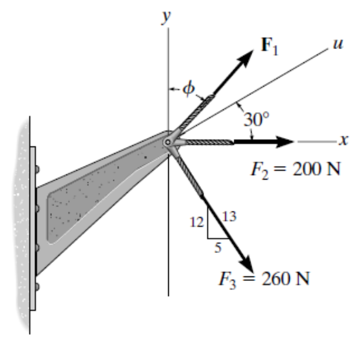
Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16° \qquad \textbf{Ans}$$



Question 4

2–51. If $F_1 = 150$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 150\sin 30° = 75\text{ N} \qquad (F_1)_y = 150\cos 30° = 129.90\text{ N}$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 75 + 200 + 100 = 375\text{ N} \rightarrow$$

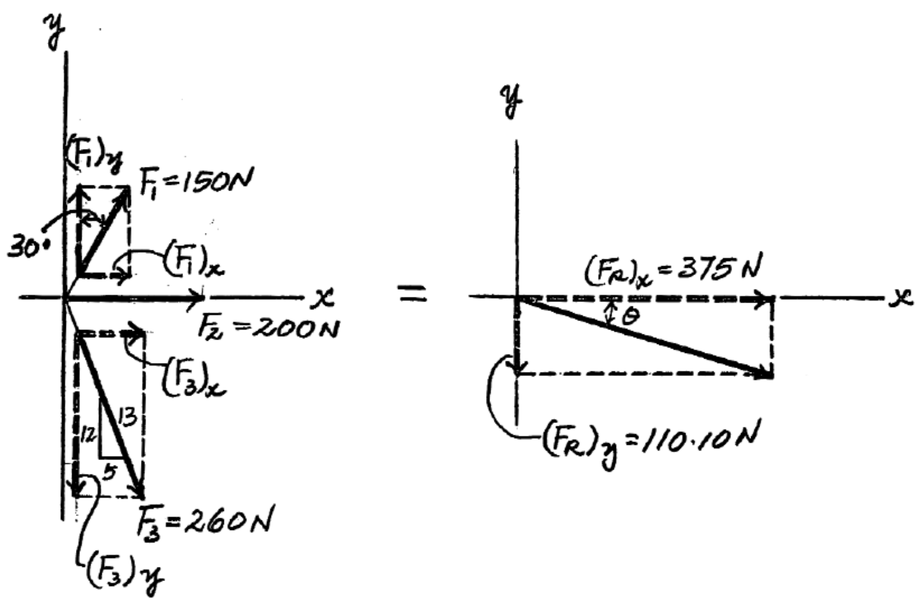
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 129.90 - 240 = -110.10\text{ N} = 110.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{375^2 + 110.10^2} = 391\text{N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

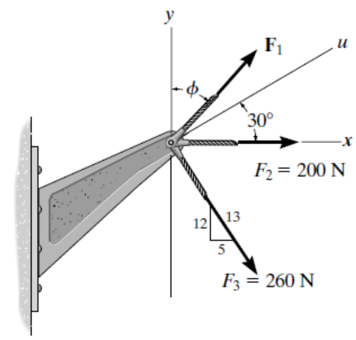
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{110.10}{375}\right) = 16.4° \qquad \textbf{Ans.}$$



(a)

Question 5

***2–52.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \sin\phi \qquad (F_1)_y = F_1 \cos\phi$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = 450\cos 30° = 389.71\text{ N} \qquad (F_R)_y = 450\sin 30° = 225\text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1\sin\phi + 200 + 100$$

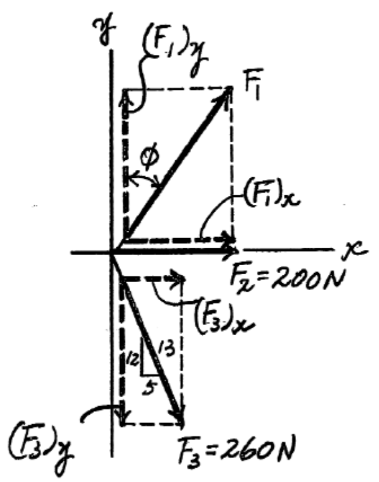
$$F_1 \sin\phi = 89.71 \qquad (1)$$

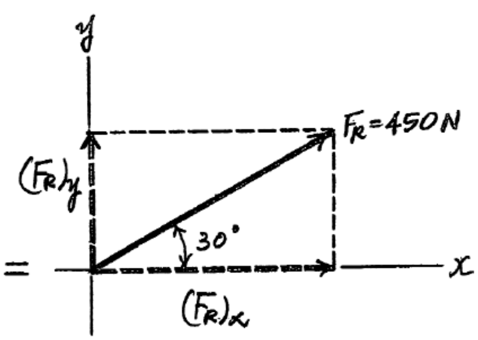
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1\cos\phi - 240$$

$$F_1\cos\phi = 465 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 10.9°$ $\qquad F_1 = 474\text{ N}$ **Ans.**

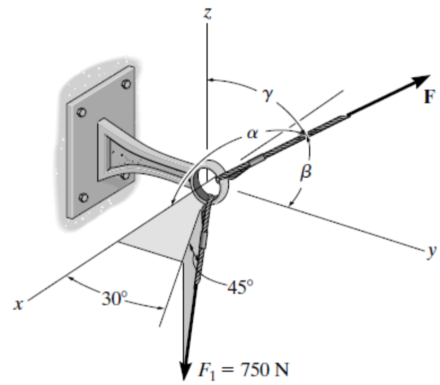




(b)

Question 6

2–70. If the resultant force acting on the bracket is to be $\mathbf{F}_R = \{800\mathbf{j}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. b and c, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45°\cos 30°(+\mathbf{i}) + 750\cos 45°\sin 30°(+\mathbf{j}) + 750\sin 45°(-\mathbf{k})$$
$$= \{459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}\}\text{N}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, Figs. a, b, and c, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$800\mathbf{j} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k})$$
$$800\mathbf{j} = (459.28 + F\cos\alpha)\mathbf{i} + (265.17 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$0 = 459.28 + F_2\cos\alpha$$
$$F\cos\alpha = -459.28 \qquad (1)$$

$$800 = 265.17 + F\cos\beta$$
$$F\cos\beta = 534.8 \qquad (2)$$

$$0 = F\cos\gamma - 530.33$$
$$F\cos\gamma = 530.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

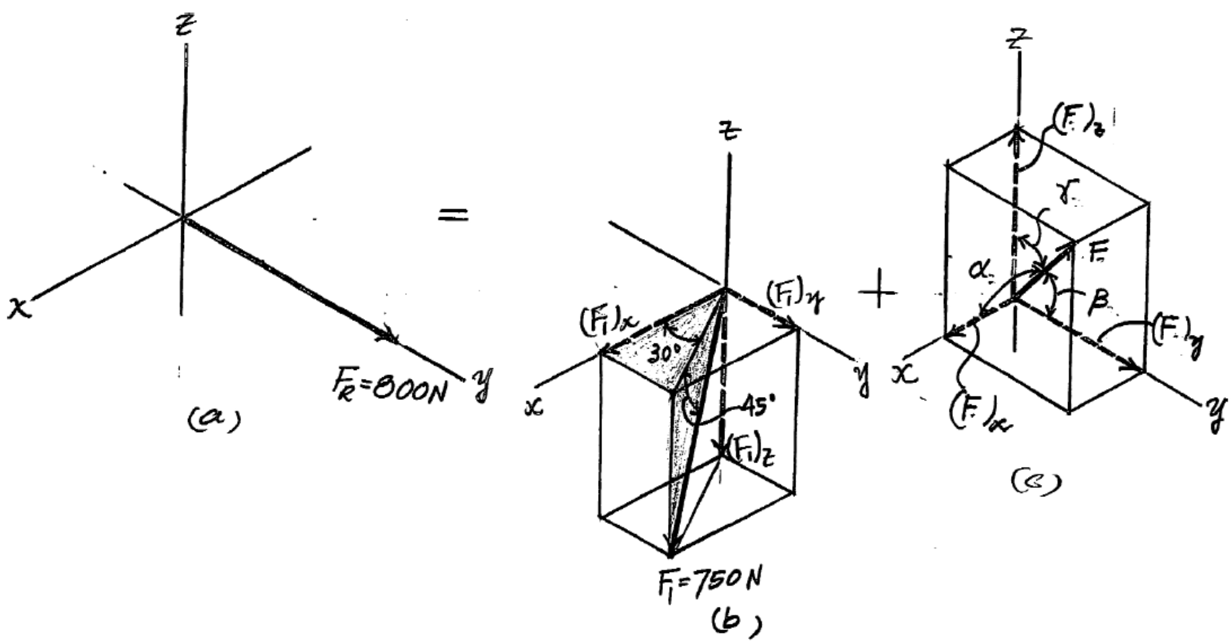
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 778\,235.93 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

$$F = 882.17\text{ N} = 882\text{ N} \qquad \textbf{Ans.}$$

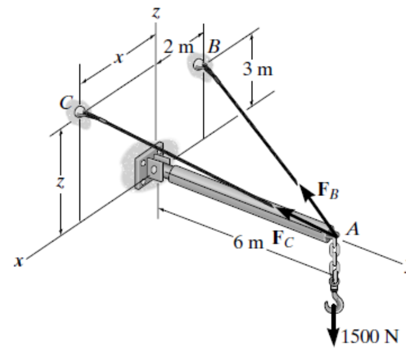
Substituting $F = 882.17$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 121° \quad \beta = 52.7° \quad \gamma = 53.0° \qquad \textbf{Ans.}$$



Question 7

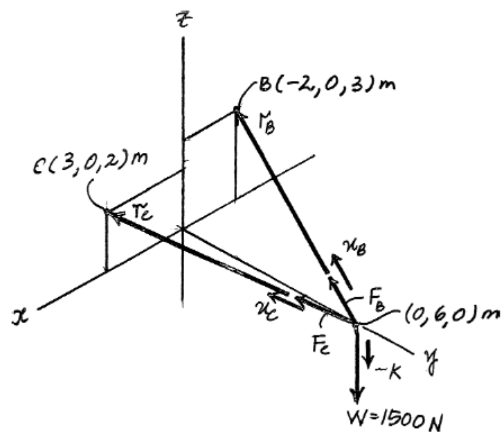
***2–100.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$



Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}) + \left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right) + (-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$z = 2.197\text{ m} = 2.20\text{ m}$ **Ans.**

Substituting $z = 2.197$ m into Eq. (4), yields

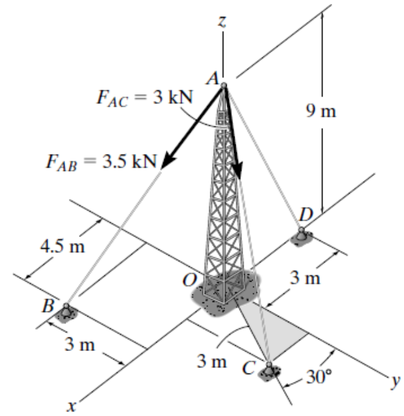
$x = 1.248\text{ m} = 1.25\text{ m}$ **Ans.**

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$F_R = 3591.85\text{ N} = 3.59\text{ kN}$ **Ans.**

Question 8

***2–120.** Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(4.5-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-9)\mathbf{k}}{\sqrt{(4.5-0)^2 + (-3-0)^2 + (0-9)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

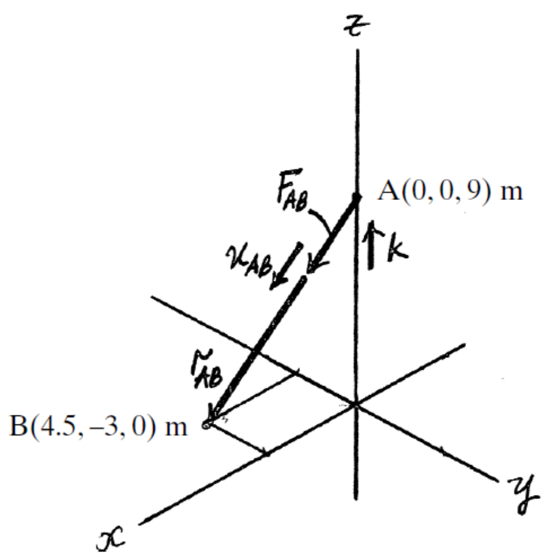
$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 3.5\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}\}\text{ kN}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AB}$ along the z axis is

$$(F_{AB})_z = \mathbf{F}_{AB} \cdot \mathbf{k} = (1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}) \cdot \mathbf{k}$$
$$= -3\text{ kN}$$

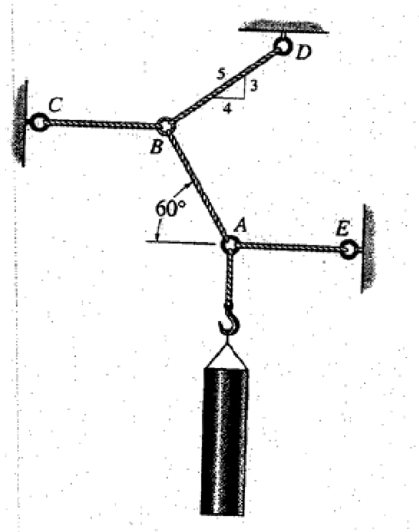
The negative sign indicates that $(F_{AB})_z$ is directed towards the negative z axis. Thus

$$(F_{AB})_z = 3\text{ kN}$$ **Ans.**



Question 9

3-39. The 30-kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.



$+\uparrow \Sigma F_y = 0;$ $T_{AB} \sin 60° - 30(9.81) = 0$

$T_{AB} = 339.83 = 340 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $T_{AE} - 339.83 \cos 60° = 0$

$T_{AE} = 170 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $T_{BD}\left(\frac{3}{5}\right) - 339.83 \sin 60° = 0$

$T_{BD} = 490.5 = 490 \text{ N}$ **Ans.**

T_{BC}

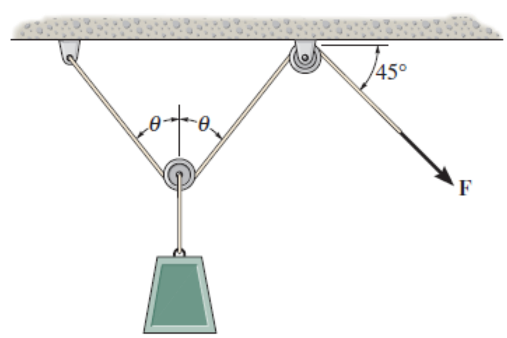
$\xrightarrow{+} \Sigma F_x = 0;$ $490.5\left(\frac{4}{5}\right) + 339.83 \cos 60° - T_{BC} = 0$

$T_{BC} = 562 \text{ N}$ **Ans.**

Question 10

3–40.

The load has a mass of 15 kg and is lifted by the pulley system shown. Determine the force **F** in the cord as a function of the angle θ. Plot the function of force F versus the angle θ for $0 \leq \theta \leq 90°$.



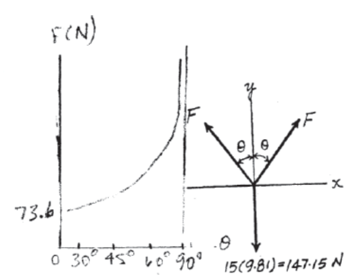
Free-Body Diagram: The tension force is the same throughout the cord.

Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;$ $F \sin \theta - F \sin \theta = 0$ *(Sati*

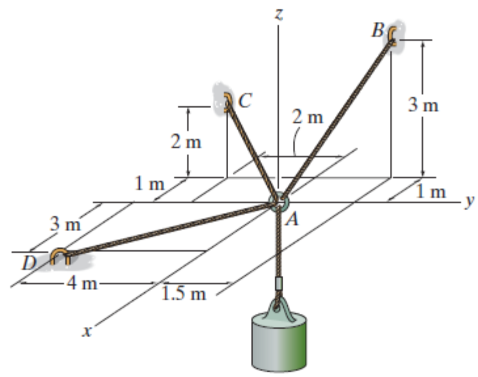
$+\uparrow \Sigma F_y = 0;$ $2F \cos \theta - 147.15 = 0$

$F = \{73.6 \sec \theta\} \text{ N}$



Question 11

3–61. Determine the tension developed in cables AB, AC, and AD required for equilibrium of the 75-kg cylinder.



Force Vectors: We can express each of the forces on the free - body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-1-0)\mathbf{i}+(1.5-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-1-0)^2+(1.5-0)^2+(3-0)^2}}\right] = -\frac{2}{7}F_{AB}\,\mathbf{i}+\frac{3}{7}F_{AB}\,\mathbf{j}+\frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-1-0)\mathbf{i}+(-2-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-1-0)^2+(-2-0)^2+(2-0)^2}}\right] = -\frac{1}{3}F_{AC}\,\mathbf{i}-\frac{2}{3}F_{AC}\,\mathbf{j}+\frac{2}{3}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(3-0)\mathbf{i}+(-4-0)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(3-0)^2+(-4-0)^2+(0-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{i}-\frac{4}{5}F_{AD}\,\mathbf{j}$$

$$\mathbf{W} = [-75(9.81)\mathbf{k}]\text{N} = [-735.75\mathbf{k}]\text{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}\mathbf{i}+\frac{3}{7}F_{AB}\mathbf{j}+\frac{6}{7}F_{AB}\mathbf{k}\right)+\left(-\frac{1}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{2}{3}F_{AC}\mathbf{k}\right)+\left(\frac{3}{5}F_{AD}\mathbf{i}-\frac{4}{5}F_{AD}\mathbf{j}\right)+(-735.75\mathbf{k}) = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD}\right)\mathbf{i}+\left(\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD}\right)\mathbf{j}+\left(\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-735.75\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD} = 0 \quad (1)$$

$$\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD} = 0 \quad (2)$$

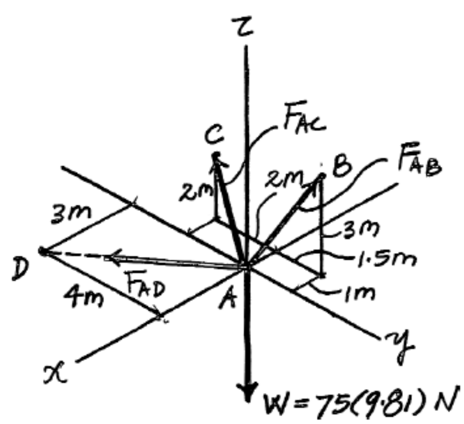
$$\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-735.75 = 0 \quad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 831\text{ N}$ **Ans.**

$F_{AC} = 35.6\text{ N}$ **Ans.**

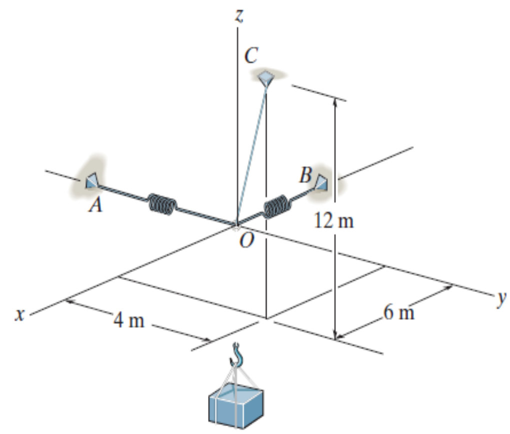
$F_{AD} = 415\text{ N}$ **Ans.**



Question 12

3–47.

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 300$ N/m.



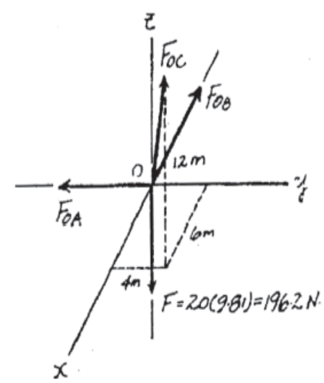
SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_{OC} = F_{OC}\left(\frac{6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}}{\sqrt{6^2 + 4^2 + 12^2}}\right) = \frac{3}{7}F_{OC}\mathbf{i} + \frac{2}{7}F_{OC}\mathbf{j} + \frac{6}{7}F_{OC}\mathbf{k}$$

$$\mathbf{F}_{OA} = -F_{OA}\mathbf{j} \qquad \mathbf{F}_{OB} = -F_{OB}\mathbf{i}$$

$$\mathbf{F} = \{-196.2\mathbf{k}\}\ \text{N}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{OC} + \mathbf{F}_{OA} + \mathbf{F}_{OB} + \mathbf{F} = \mathbf{0}$$

$$\left(\frac{3}{7}F_{OC} - F_{OB}\right)\mathbf{i} + \left(\frac{2}{7}F_{OC} - F_{OA}\right)\mathbf{j} + \left(\frac{6}{7}F_{OC} - 196.2\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$\frac{3}{7}F_{OC} - F_{OB} = 0 \qquad \textbf{(1)}$$

$$\frac{2}{7}F_{OC} - F_{OA} = 0 \qquad \textbf{(2)}$$

$$\frac{6}{7}F_{OC} - 196.2 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1),(2) and (3) yields

$$F_{OC} = 228.9\ \text{N} \qquad F_{OB} = 98.1\ \text{N} \qquad F_{OA} = 65.4\ \text{N}$$

Spring Elongation: Using spring formula, Eq. 3–2, the spring elongation is $s = \dfrac{F}{k}$.

$$s_{OB} = \frac{98.1}{300} = 0.327\ \text{m} = 327\ \text{mm} \qquad \textbf{Ans.}$$

$$s_{OA} = \frac{65.4}{300} = 0.218\ \text{m} = 218\ \text{mm} \qquad \textbf{Ans.}$$