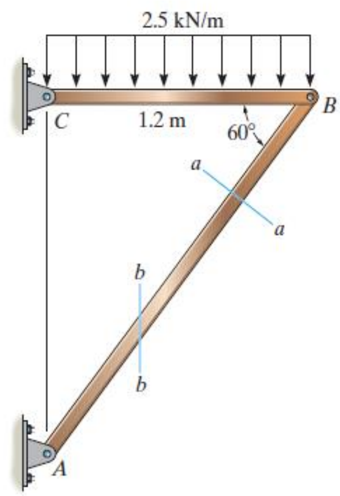
BYG140 KONSTRUKSJONSMEKANIKK 1

Assignment (4) Solutions

(Mechanics of Materials Ch 1: Stress, Ch 2: Strain, Ch 3: Mechanical Properties of Materials, Ch 4: Axial Load and Ch 6:Bending)

Question 1

Determine the average normal stress at section a–a and the average shear stress at section b–b in member AB. The cross section is square, 12 mm on each side.



Consider the FBD of member BC, Fig. a,

$$\zeta + \Sigma M_C = 0; \quad F_{AB}\sin 60°(1.2) - 2.5(1.2)(0.6) = 0 \quad F_{AB} = 1.732\text{ kN}$$

Referring to the FBD in Fig. b,

$$^{+}\swarrow \Sigma F_{x'} = 0; \quad N_{a\text{–}a} + 1.732 = 0 \qquad N_{a\text{–}a} = -1.732\text{ kN}$$

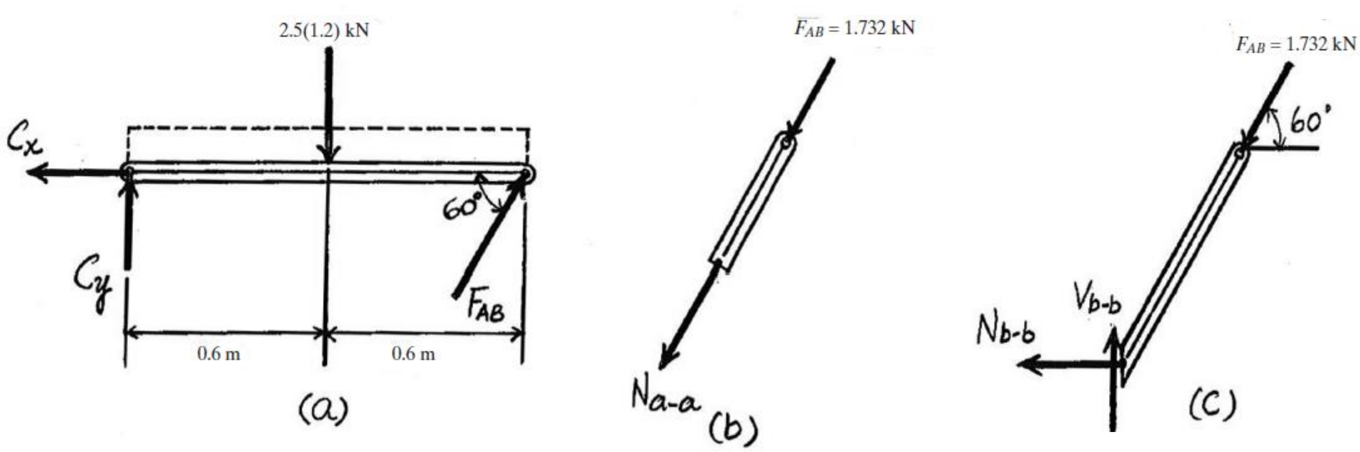
Referring to the FBD in Fig. c.

$$+\uparrow \Sigma F_y = 0; \quad V_{b\text{–}b} - 1.732\sin 60° = 0 \qquad V_{b\text{–}b} = 1.500\text{ kN}$$

The cross-sectional areas of section a–a and b–b are $A_{a\text{–}a} = 12(12) = 144\text{ mm}^2$ and $A_{b-b} = 12\left(\dfrac{12}{\cos 60°}\right) = 288\text{ mm}^2$. Thus

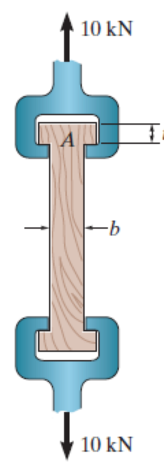
$$\sigma_{a-a} = \frac{N_{a-a}}{A_{a-a}} = \frac{1.732(10^3)}{144} = 12.0\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{b-b} = \frac{V_{b-b}}{A_{b-b}} = \frac{1.500(10^3)}{288} = 5.21\text{ MPa} \qquad \textbf{Ans.}$$



Question 2

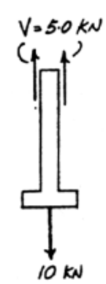
The wood specimen is subjected to the pull of 10 kN in a tension testing machine. If the allowable normal stress for the wood is $(\sigma_t)_{allow} = 12$ MPa and the allowable shear stress is $\tau_{allow} = 1.2$ MPa, determine the required dimensions b and t so that the specimen reaches these stresses simultaneously. The specimen has a width of 25 mm.



Allowable Shear Stress: Shear limitation

$$\tau_{allow} = \frac{V}{A}; \qquad 1.2\left(10^6\right) = \frac{5.00(10^3)}{(0.025)\, t}$$

$$t = 0.1667 \text{ m} = 167 \text{ mm}$$



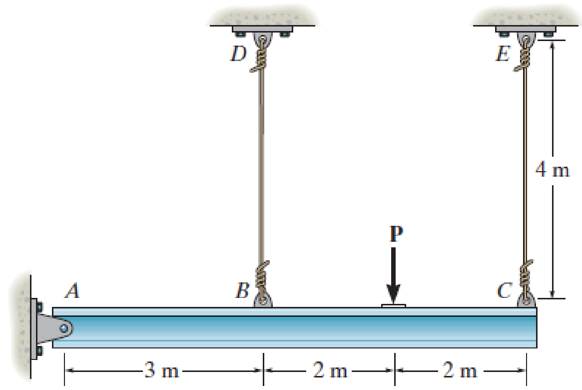
Allowable Normal Stress: Tension limitation

$$\sigma_{allow} = \frac{P}{A}; \qquad 12.0\left(10^6\right) = \frac{10(10^3)}{(0.025)\, b}$$

$$b = 0.03333 \text{ m} = 33.3 \text{ mm}$$

Question 3

The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.

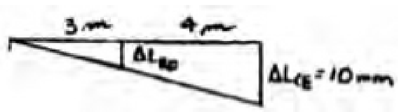


$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

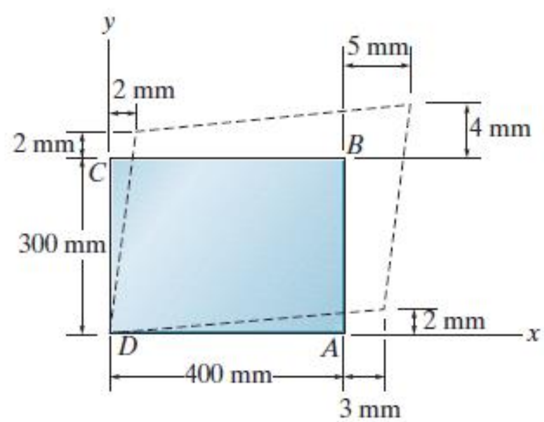
$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$

$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$



Question 4

The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$

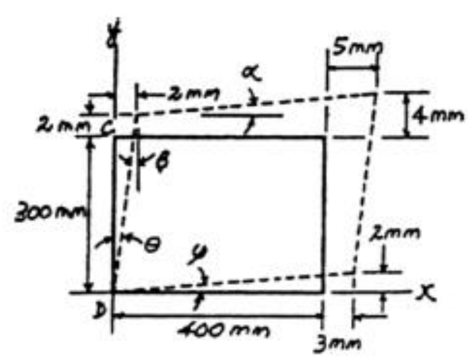
Shear Strain:

$$(\gamma_C)_{xy} = -(\alpha + \beta)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad}$$

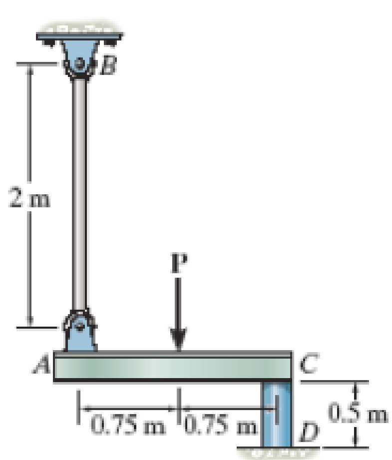
$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$



Question 5

The stress-strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut *AB* and post *CD* made from this material, determine the largest load *P* that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.



Given:

$L_{AB} := 2m$ $\quad L_{CD} := 0.5m$ $\quad L_{AC} := 1.5m$

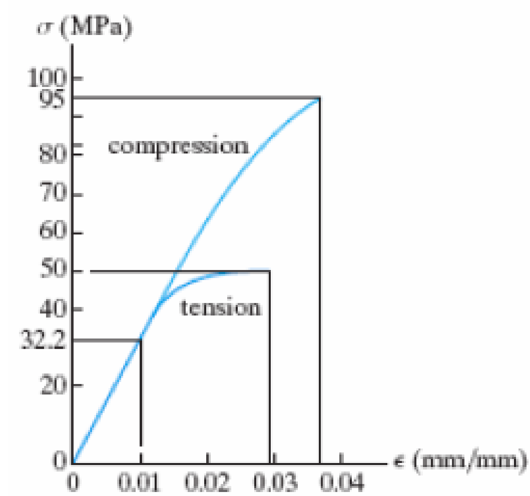
$d_{AB} := 12mm$ $\quad d_{CD} := 40mm$

Solution:

Support Reactions: $A_y = \frac{P}{2}$ $\quad C_y = \frac{P}{2}$

$F_{AB} = A_y$ $\quad F_{CD} = C_y$

$Area_{AB} := \left(\frac{\pi}{4}\right)\cdot d_{AB}^{\,2}$ $\quad Area_{CD} := \left(\frac{\pi}{4}\right)\cdot d_{CD}^{\,2}$



For rupture of strut *AB*:

From the stress-strain diagram, $\sigma_{R\_t} := 50.0MPa$

$F_{AB} = Area_{AB}\cdot\left(\sigma_{R\_t}\right)$

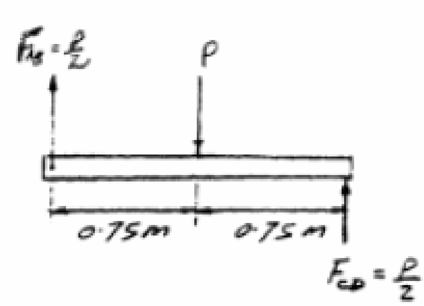
$P := 2Area_{AB}\cdot\left(\sigma_{R\_t}\right)$

$P = 11.31\,kN$ **Ans**

(Controls!)

For rupture of post *CD*:

From the stress-strain diagram, $\sigma_{R\_c} := 95.0MPa$



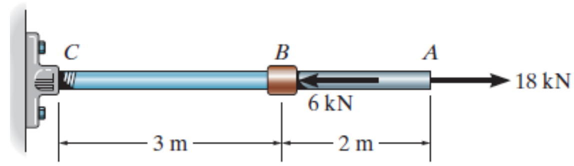
$F_{CD} = Area_{CD}\cdot\left(\sigma_{R\_c}\right)$

$P := 2Area_{CD}\cdot\left(\sigma_{R\_c}\right)$

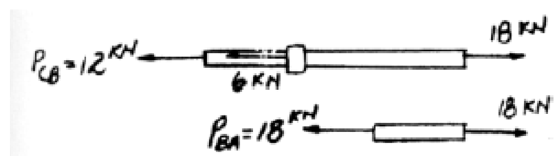
$P = 238.76\,kN$

Question 6

The assembly consists of a steel rod CB and an aluminum rod BA, each having a diameter of 12 mm. If the rod is subjected to the axial loadings at A and at the coupling B, determine the displacement of the coupling B and the end A. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at B and C, and assume that they are rigid. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



$$\delta_B = \frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} = 0.00159 \text{ m} = 1.59 \text{ mm}$$

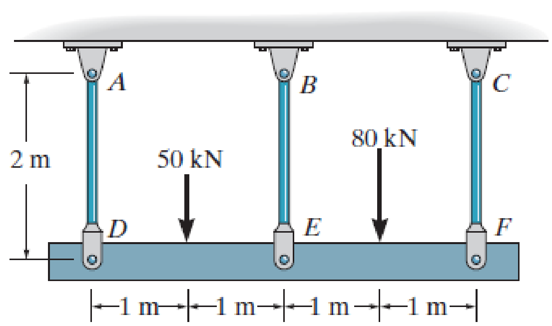


$$\delta_A = \Sigma\frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} + \frac{18(10^3)(2)}{\frac{\pi}{4}(0.012)^2(70)(10^9)}$$

$$= 0.00614 \text{ m} = 6.14 \text{ mm}$$

Question 7

The three suspender bars are made of A992 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



Referring to the *FBD* of the rigid beam, Fig. *a*,

$+\uparrow\Sigma F_y = 0;\quad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0$ (1)

$\circlearrowleft +\Sigma M_D = 0;\quad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0$ (2)

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\,\not{L}}{\not{A}\not{E}} = \frac{1}{2}\left(\frac{F_{AD}\not{L}}{\not{A}\not{E}} + \frac{F_{CF}\,\not{L}}{\not{A}\not{E}}\right)$$

$F_{AD} + F_{CF} = 2\,F_{BE}$ (3)

Solving Eqs. (1), (2), and (3) yields

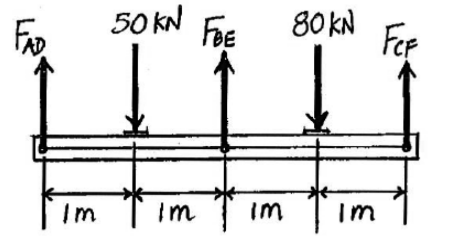
$$F_{BE} = 43.33(10^3)\text{ N} \qquad F_{AD} = 35.83(10^3)\text{ N} \qquad F_{CF} = 50.83(10^3)\text{ N}$$

Thus,

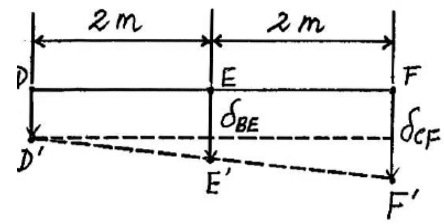
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{ MPa}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{ MPa}$$

$$\sigma_{CF} = 113\text{ MPa}$$



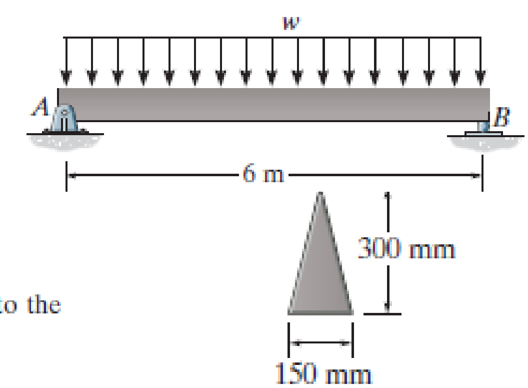
(a)



(b)

Question 8

If the material of the beam has an allowable bending stress of $\sigma_{allow} = 150$ MPa, determine the maximum allowable intensity w of the uniform distributed load.

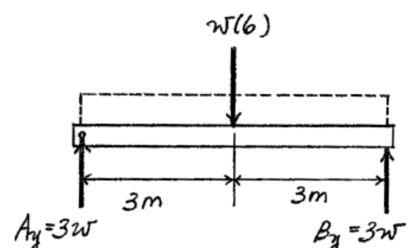


Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*,

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad 3w - wx = 0 \qquad x = 3 \text{ m}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M_{max} + w(3)\left(\frac{3}{2}\right) - 3w(3) = 0 \qquad M_{max} = \frac{9}{2}w$$



(a)

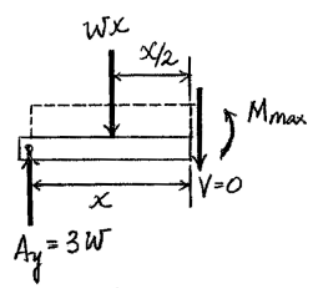
Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3})\text{ m}^4$$

Absolute Maximum Bending Stress: Here, $c = \frac{2}{3}(0.3) = 0.2$ m.

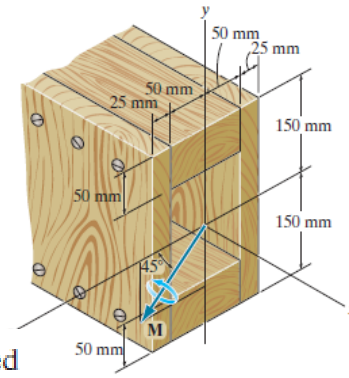
$$\sigma_{allow} = \frac{Mc}{I}; \qquad 150(10^6) = \frac{\frac{9}{2}w(0.2)}{0.1125(10^{-3})}$$

$$w = 18750 \text{ N/m} = 18.75 \text{ kN/m}$$ **Ans.**



(b)

Question 9

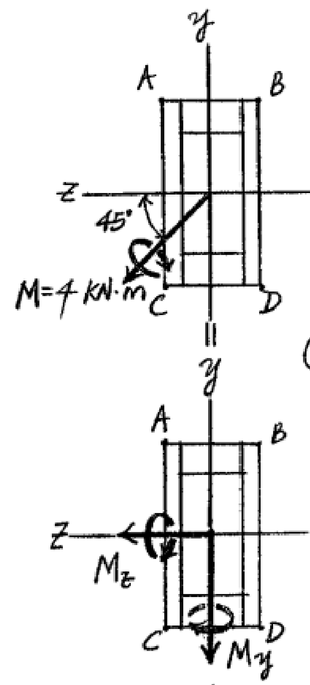


The box beam is subjected to the internal moment of $M = 4$ kN · m, which is directed as shown. Determine the maximum bending stress developed in the beam and the orientation of the neutral axis.

Internal Moment Components: The y component of **M** is negative since it is directed towards the negative sense of the y axis, whereas the z component of **M** which is directed towards the positive sense of the z axis is positive, Fig. *a*. Thus,

$$M_y = -4 \sin 45^\circ = -2.828 \text{ kN} \cdot \text{m}$$

$$M_z = 4 \cos 45^\circ = 2.828 \text{ kN} \cdot \text{m}$$



(a)

Section Properties: The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.3)(0.15^3) - \frac{1}{12}(0.2)(0.1^3) = 67.7083(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.15)(0.3^3) - \frac{1}{12}(0.1)(0.2^3) = 0.2708(10^{-3}) \text{ m}^4$$

Bending Stress: By inspection, the maximum bending stress occurs at corners A and D.

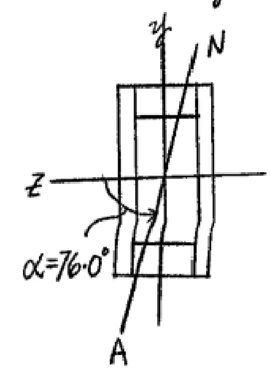
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_{max} = \sigma_A = -\frac{2.828(10^3)(0.15)}{0.2708(10^{-3})} + \frac{(-2.828)(10^3)(0.075)}{67.7083(10^{-6})}$$

$$= -4.70 \text{ MPa} = 4.70 \text{ MPa (C)} \qquad \textbf{Ans.}$$

$$\sigma_{max} = \sigma_D = -\frac{2.828(10^3)(-0.15)}{0.2708(10^{-3})} + \frac{(-2.828)(10^3)(-0.075)}{67.7083(10^{-6})}$$

$$= 4.70 \text{ MPa (T)} \qquad \textbf{Ans.}$$



(b)

Orientation of Neutral Axis: Here, $\theta = -45^\circ$.

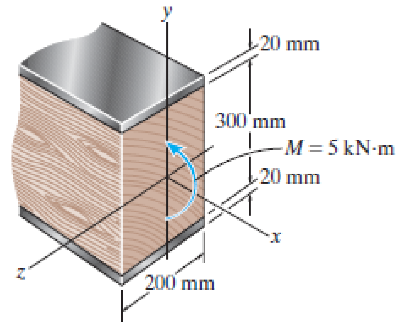
$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

$$\tan \alpha = \frac{0.2708(10^{-3})}{67.7083(10^{-6})} \tan(-45^\circ)$$

$$\alpha = -76.0^\circ \qquad \textbf{Ans.}$$

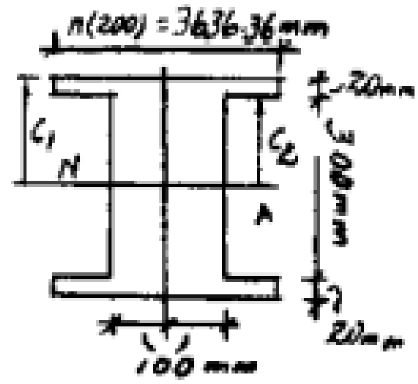
The orientation of the neutral axis is shown in Fig. *b*.

Question 10



A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a bending moment of $M = 5\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section. Take $E_w = 11$ GPa, $E_{st} = 200$ GPa.

$$n = \frac{200}{11} = 18.182$$

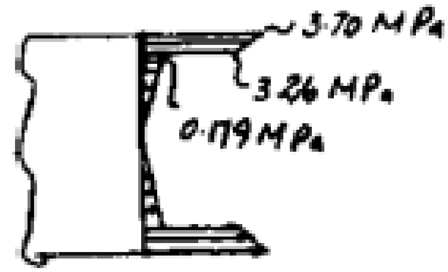


$$I = \frac{1}{12}(3.63636)(0.34)^3 - \frac{1}{12}(3.43636)(0.3)^3 = 4.17848(10^{-3})\ \text{m}^4$$

Maximum stress in steel:

$$(\sigma_{st})_{max} = \frac{nMc_1}{I} = \frac{18.182(5)(10^3)(0.17)}{4.17848(10^{-3})} = 3.70\ \text{MPa}$$

Maximum stress in wood:



$$(\sigma_w)_{max} = \frac{Mc_2}{I} = \frac{5(10^3)(0.15)}{4.17848(10^{-3})} = 0.179\ \text{MPa}$$

$$(\sigma_{st}) = n(\sigma_w)_{max} = 18.182(0.179) = 3.26\ \text{MPa}$$