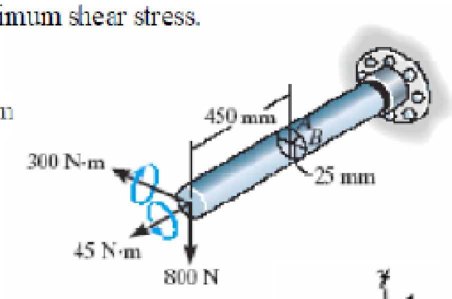
BYG140 KONSTRUKSJONSMEKANIKK 1

Assignment (8) - Solutions

Question 1

The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at points *A* and *B* and the absolute maximum shear stress.



Given: $r_o := 25\text{mm}$ $\quad L := 450\text{mm}$

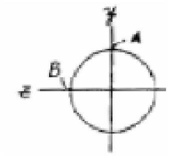
$P := 0.8\text{kN}$ $\quad T_o := 0.045\text{kN}\cdot\text{m}$ $\quad M_o := 0.3\text{kN}\cdot\text{m}$

Solution: $\rho := r_o$

Internal Force and Moment : At Section *AB*:

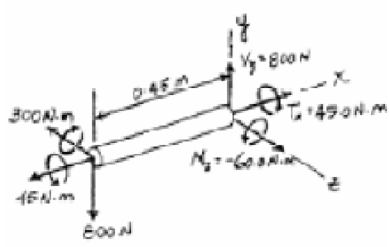
$$V_y := P \qquad T_x := T_o \qquad M_z := M_o - P\cdot L$$

Section Property :



$$A := \pi\cdot r_o^{\,2} \qquad I_z := \frac{\pi}{4}\cdot r_o^{\,4} \qquad J := \frac{\pi}{2}\cdot r_o^{\,4}$$

$$Q_A := 0 \quad (\text{Since } A'=0) \qquad Q_B := \frac{4\cdot r_o}{3\cdot\pi}\cdot(0.5A)$$

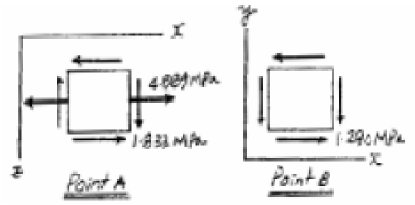


Normal Stress: $c_A := r_o \qquad c_B := 0$

$$\sigma_A := -\frac{M_z\cdot c_A}{I_z} \qquad \sigma_A = 4.889\,\text{MPa}$$

$$\sigma_B := -\frac{M_z\cdot c_B}{I_z} \qquad \sigma_B = 0\,\text{MPa}$$

Shear Stress : $b_B := 2\cdot r_o$

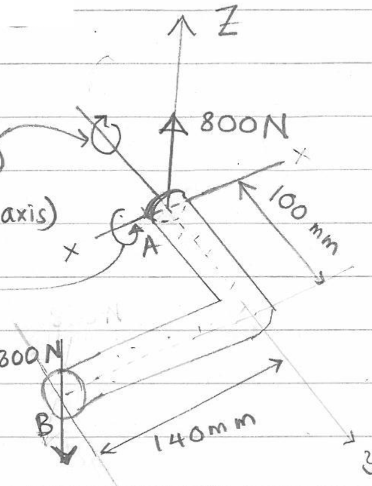


$$\tau_A := \frac{T_x\cdot\rho}{J} \qquad \tau_A = 1.833\,\text{MPa}$$

$$\tau_B := \frac{V_y\cdot Q_B}{I_z\cdot b_B} - \frac{T_x\cdot\rho}{J} \qquad \tau_B = -1.290\,\text{MPa}$$

Question 2

(a)

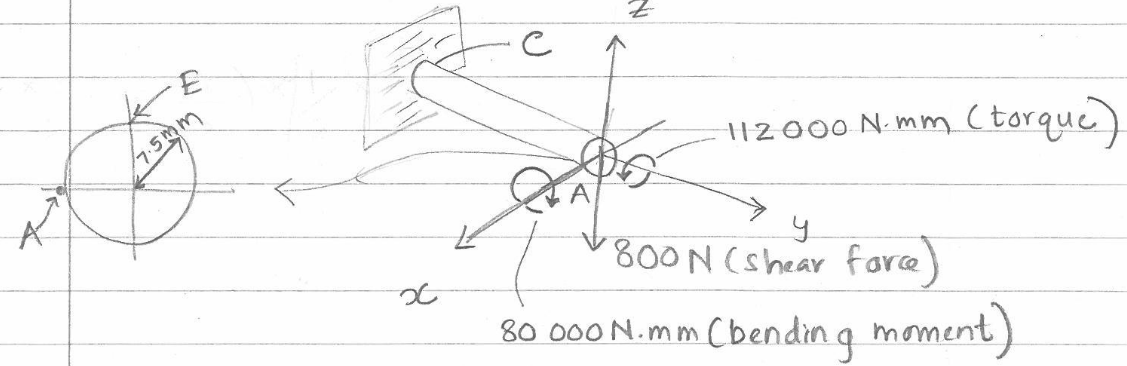


$800N \times (140mm) = 112\,000$ N.mm (Torque about y axis)

$800N \times (100mm) = 80\,000$ N.mm (bending moment about x axis)

Consider the equilibrium of segment 'AB'

Considering "BC" segment,



(b) Stress at point A'

Shear, $\rightarrow \tau_{y,z} = \dfrac{VQ}{It}$

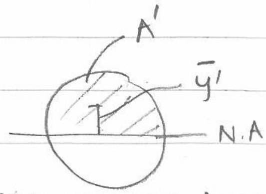
$V = 800N$, $Q = \bar{y}' A' = \dfrac{4 \times (7.5mm)}{3\pi}\left[\dfrac{\pi \times 7.5mm^2}{2}\right]$

$= 281.25\ mm^3$

$I = \frac{1}{4}\pi (7.5mm)^4 \qquad t = 2 \times 7.5\ mm$

$\tau_{y,z} = \dfrac{(800N)(281.25\ mm^3)}{\left[\frac{1}{4}\pi (7.5mm)^4\right] 2 \times 7.5mm}$

$= \underline{6.04}$ MPa



• Use attached Equations of statics.

$\bar{y}' = \dfrac{4r}{3\pi}$

$A' = \pi r^2/2$

Shear due to torque

$$\tau_{yz} = \frac{T\rho}{J}$$

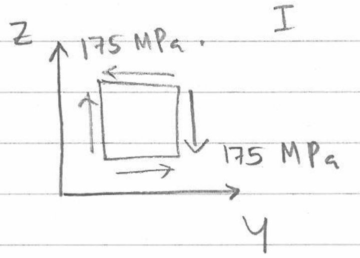
$T = 112000 \text{ N.mm}$, $\rho = 7.5 \text{ mm}$ $J = \left(\frac{1}{2}\pi (7.5\text{mm})^4\right)$

$$\tau_{yz} = \frac{112\,000 \text{ N.mm } (7.5\text{mm})}{\left[\frac{1}{2}\pi \,(7.5\text{mm})^4\right]}$$

$$= \underline{\underline{169 \text{ MPa}}}$$

Bending

$$\sigma_A = \frac{My}{I} \qquad y = 0, \quad \sigma_A = 0.$$

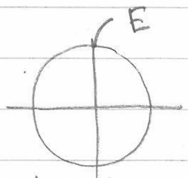


Total shear stress at A. $= 169 + 6.04$

$= \underline{\underline{175 \text{ MPa}}}$

Normal stress $= 0$ //

Stress at point E



Shear stress due to shear force (800N)

shear stress due to shear force $= 0$ // $(\bar{y}' = 0)$

Shear stress due torque $= \tau_{xy} = \frac{T\rho}{J} = 169 \text{ MPa}$ //

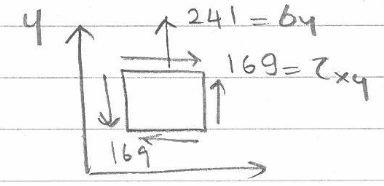
because, $T = 112000 \text{ N.mm}$, $\rho = 7.5 \text{ mm}$,
$J = \frac{1}{2}\pi\,(7.5\text{mm})^4$.

Bending stress, $\sigma_E = \frac{M}{I} y$

$M = 80\,000\,N.mm$, $I = \frac{1}{4}\pi \times (7.5\,mm)^4$. $y = 7.5\,mm$.

$$\sigma_E = \frac{80\,000\,N.mm \times 7.5\,mm}{\frac{1}{4}\pi \times (7.5\,mm)^4}$$

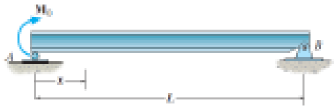
$$= \underline{241\,MPa}$$



State of stress at E

Question 3

Determine the equations of the elastic curve for the beam using the x coordinate. Specify the slope at A and maximum deflection. EI is constant.



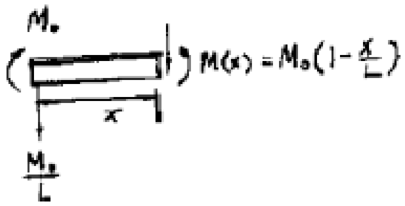
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0\left(1 - \frac{x}{L}\right)$$

$$EI\frac{dv}{dx} = M_0\left(x - \frac{x^2}{2L}\right) + C_1 \qquad (1)$$

$$EIv = M_0\left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + C_1x + C_2 \qquad (2)$$

Boundary Conditions:



$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0\left(\frac{L^2}{2} - \frac{L^2}{6}\right) + C_1L; \quad C_1 = -\frac{M_0L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}\left(x - \frac{x^2}{2L} - \frac{L}{3}\right) \qquad (3)$$

$$\theta_A = \frac{dv}{dx}\bigg|_{x=0} = -\frac{M_0L}{3EI} \qquad \textbf{Ans.}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}\left(x - \frac{x^2}{2L} - \frac{L}{3}\right)$$

$3x^2 - 6Lx + 2L^2 = 0; \qquad x = 0.42265\,L$

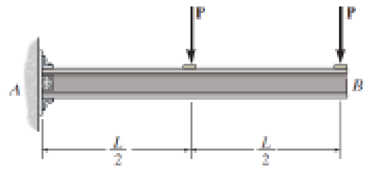
$$v = \frac{M_0}{6EIL}(3Lx^2 - x^3 - 2L^2x) \qquad (4) \qquad \textbf{Ans.}$$

Substitute x into v,

$$v_{max} = \frac{-0.0642M_0L^2}{EI} \qquad \textbf{Ans.}$$

Question 4

Determine the deflection of end B of the cantilever beam. EI is constant.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

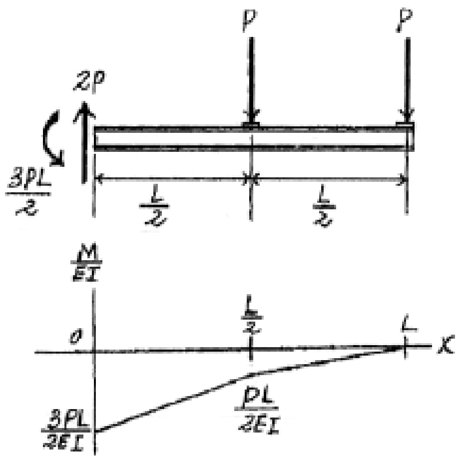
Moment Area Theorem. Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_B = |\theta_{B/A}| = \frac{1}{2}\left[\frac{3PL}{2EI} + \frac{PL}{2EI}\right]\left(\frac{L}{2}\right) + \frac{1}{2}\left[\frac{PL}{2EI}\right]\left(\frac{L}{2}\right)$$

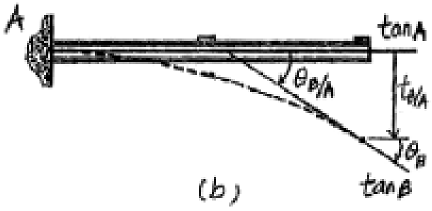
$$= \frac{5PL^2}{8\,EI} \circlearrowright \qquad \textbf{Ans.}$$

$$\Delta_B = |t_{B/A}| = \left(\frac{3L}{4}\right)\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{5L}{6}\left[\frac{1}{2}\left(\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\right] + \frac{L}{3}\left[\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{7PL^3}{16EI} \downarrow \qquad \textbf{Ans.}$$



(a)



(b)

Question 5

Determine the deflection at C and the slope of the beam at A, B, and C. EI is constant.

$$t_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(2) = \frac{-48}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(3+2) + \left(\frac{-8}{EI}\right)(3)(1.5) = \frac{-156}{EI}$$

$$\Delta_C = |t_{C/A}| - \frac{9}{6}|t_{B/A}| = \frac{156}{EI} - \frac{9(48)}{6(EI)} = \frac{84}{EI}$$

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{8}{EI}$$

$$\theta_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) = \frac{-24}{EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = -\frac{24}{EI} + \frac{8}{EI} = -\frac{16}{EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) + \left(\frac{-8}{EI}\right)(3) = \frac{-48}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = -\frac{48}{EI} + \frac{8}{EI} = -\frac{40}{EI}$$