Høsten 2020

FYS100 Fysikk: Resit Exam/Konte-Eksamen: Solutions

You **must** put your candidate number on every sheet.

There are 4 questions. You need to answer all 4 questions for a full score.

The standard formula sheet for FYS100 Fysikk is part of this question sheet.

Standard approved calculators are allowed.

Don't panic! Draw a diagram where relevant. State clearly the relevant physics. Show your working.

The questions are also attached in Norwegian.

Good luck!

Du **må** legge kandidatnummeret ditt på hvert ark.

Det er 4 spørsmål. Du må svare på alle de fire spørsmålene for en full score.

Standardformelarket for FYS100 Fysikk er en del av dette spørsmålet.

Standard godkjente kalkulatorer er tillatt.

Ingen panikk! Tegn et diagram der det er relevant. Angi tydelig hvilken fysikk som er relevant. Vis arbeidet ditt.

Spørsmålene er også vedlagt på engelsk.

Lykke til!

Problem 1: Moving a block

A man of mass $m_1 = 84$kg wants to move a rectangular block of mass $M = 110$kg on a flat surface. The coefficient of static friction between the block and the surface is $\mu_s = 0.80$. The coefficient of kinetic friction between the block and the surfaces is $\mu_k = 0.72$.

a) Initially a child of mass $m_2 = 27$kg sits on top of the block. If the man pushes horizontally on the block with a force of $F_h = 990$N, show whether the block will move or not.

Solution: In order to move the block the man must overcome the maximum static friction force $F_{s,max}$. This is given by $F_{s,max} = \mu_s N$ where N is the normal force from the ground. To find the normal force we must use Newton's laws and resolve vertical forces. There are three vertical forces acting on the block, its mass acting down, the normal force from the child acting down and the normal force from the ground acting up. Neither the child nor the block accelerates vertically so $(m_c + m_b)g = N$. So the maximum static friction is

$$F_{s,max} = \mu_s(m_c + m_b)g = 0.8 * (27 + 110) * 9.8 = 1076\text{N} \quad (1)$$

This is more than the pushing force applied by the man (990N), so the block doesn't move.

b) What is the value of the frictional force on the block when the man pushes on it?

Solution: Since the block does not move horizontally, the horizontal forces must balance by Newton's laws. There are only two horizontal forces on the block, the push from the man and the static friction force. If the man pushes with a horizontal force of 990N then the static friction force is also 990N exactly.

c) If the child now jumps off the block, what will be the acceleration of the block when the man pushes on it?

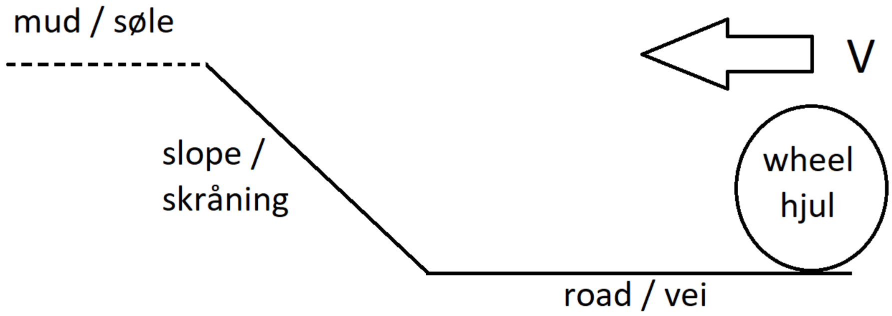
Solution: Now the normal force from the child on the block is removed. The maximum static friction force is $F_{s,max} = \mu_s(m_b)g = 0.8 * 110 * 9.8 = 864$N so the block will move. Once the block is moving, the friction force is kinetic. By Newton's 2nd law the horizontal acceleration of the block will be

$$a = \frac{F_{push} - \mu_k m_b g}{m_b} = \frac{990 - 0.72 * 110 * 9.8}{110} = 1.93 = 1.9\text{ms}^{-2}(\text{2sig.fig.}) \quad (2)$$

d) If the coefficients of friction between the man's feet and the surface are the same as between the block and the surface, does the man slip when pushing the block? Is the man able to move the block while slipping and applying a horizontal force of 990N on the block?

Solution: The maximum static friction force on the man is $F_{s,max,man} = \mu_s m_m g = 0.8 * 84 * 9.8 = 660\text{N}$. So the man will slip. He will still be able to move the block, because he still applies a force of 990N on the block and the block applies a force of 990N on him (which causes him to slip) by Newton's third law.

Problem 2: A rolling wheel



A wheel of mass $m = 25\text{kg}$ and radius $R = 0.27\text{m}$ is rolling along a road without slipping.

a) If the wheel is moving with a speed of 89km/h, what is the angular velocity of the wheel?

Solution: The speed of the wheel in m/s is $89 * 1000/3600 = 24.7$m/s. The angular velocity is given by $\omega = \frac{v}{r} = \frac{89*1000}{3600*0.27} = 91.56 = 92\text{s}^{-1}$ to 2 sig. fig.

b) It takes 0.69 seconds for the wheel to make ten complete revolutions as it rolls along the road. What constant angular acceleration would the wheel have if its initial velocity is 89km/h?

Solution: Use the kinetic relation for a constant angular acceleration $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$. For ten complete revolutions the wheel turns

through $10 * 2 * \pi$ **radians. This gives**

$$\alpha = \frac{(\theta_f - \theta_i) - \omega_i t}{\frac{1}{2}t^2} = \frac{10 * 2 * \pi - \frac{89*1000}{3600*0.27} * 0.69}{\frac{1}{2}0.69^2} = -1.458 = -1.5\text{s}^{-2}\,2\,\text{sig. fig.} \tag{3}$$

The angular acceleration is negative because the wheel is decelerating (points in opposite direction to angular velocity.)

c) At the end of the road is a slope of constant angle θ to the horizontal and total length along the slope L. Show that in order for the wheel to reach the top of the slope without slipping, the translational velocity of the wheel at the bottom of the slope must be at least v_{min}, where

$$v_{\text{min}} = R\sqrt{\frac{2mgL\sin\theta}{(I + mR^2)}}, \tag{4}$$

g is the acceleration due to gravity and I is the moment of inertia of the wheel. List any assumptions or approximations you make in deriving this result.

Solution: Assume conservation of energy. Assume that the initial energy of the wheel is given by lateral kinetic energy $\frac{1}{2}mv^2$ **and rotational kinetic energy** $\frac{1}{2}I\omega^2$**. If the wheel just comes to a stop at the top of the slope, the final energy is just gravitational potential energy,** mgh**. So assuming there are no other sources of energy or energy lost, we have**

$$mgh = \frac{1}{2}mv_{min}^2 + \frac{1}{2}I\omega^2 \tag{5}$$

The height h **of the slope is given by trigonometry as** $h = L\sin\theta$**. Using the relation for a non-slipping wheel** $v = \omega r$ **and rearranging the above algebraically, we obtain the result.**

d) The wheel can be modelled as a solid cylinder with moment of inertia around its central axis of $I = \frac{1}{2}mR^2$. At the top of the slope, there is a very muddy field, where rolling resistance applies a constant torque on the wheel of $\tau = 19$Nm. If the wheel comes to a stop in $d = 2.9$m without slipping, calculate what its initial translational velocity was at the top of the slope before the muddy field.

Solution: By the angular version of Newton's second law $\tau = I\alpha$ **we know that the torque from the mud gives rise to an angular acceleration. The distance travelled by the wheel (if it doesn't slip) corresponds to moving through an angle of** d/R **in radians. Putting**

it into the kinetic relation for the angle turned $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ we have

$$v_i = R\sqrt{2\frac{\tau}{\frac{1}{2}mR^2}\frac{d}{R}} = \sqrt{\frac{4*19*2.9}{25*0.27}} = 5.714 = 5.7\text{ms}^{-1} \quad (6)$$

Problem 3: Planetary collision

A solid spherical planet has a uniform density of $\rho = 5500\text{kgm}^{-3}$, a radius of $R = 2300\text{km}$ and a rotational period of $T = 11$ hours.

a) What is the magnitude of the angular velocity of the planet around its axis of rotation?

Solution: The angular velocity is given by $\omega = \frac{2\pi}{T} = \frac{2\pi}{11*60*60} = 0.0001587 = 0.00016\text{s}^{-1}$ **to 2 sig. fig.**

b) Looking along the rotational axis of the planet in the direction of the angular velocity, does the planet rotate clockwise or anti-clockwise?

Solution: By the right-hand rule, if we look in the direction of the angular velocity, the planet will be rotating clockwise.

c) Using the moment of inertia of a solid sphere of $I = \frac{2}{5}MR^2$, calculate the magnitude of the angular momentum of the planet.

Solution: The angular momentum for a sphere through its centre is given by $L = I\omega = \frac{2}{5}\rho\frac{4}{3}\pi R^3 R^2 \frac{2\pi}{T} = \frac{16*5500*\pi^2}{15*11*60*60}(2300,000)^5 = 9.4 \times 10^{31}$ **kgm^{2}s^{-1}.**

d) A large asteroid now collides with the planet. In an x, y, z coordinate system (with z the direction of the planet's angular momentum calculated above) the asteroid has total angular momentum components of $(8.9 \times 10^{30}, 3.8 \times 10^{30}, -2.3 \times 10^{31})$ in units of kg m^{2}s^{-1}. Assuming the asteroid is completely absorbed by the planet in a perfectly inelastic collision, what is the magnitude of the total angular momentum of the planet plus asteroid after the collision?

Solution: We need to use conservation of angular momentum and add the initial angular momenta. The initial momentum components of the planet are $(0, 0, 9.4\times10^{31})$ **so the sum is** $(8.9\times10^{30}, 3.8\times10^{30}, 7.1\times10^{31})$**. To get the magnitude, we need to sum the squares of the components and take the square root.** $\sqrt{(8.9 \times 10^{30})^2 + (3.8 \times 10^{30})^2 + (7.1 \times 10^{31})^2} = 7.3 \times 10^{31}$**kg m^{2}s^{-1}.**

Problem 4: Hydroelectric power and cars

A hydroelectric reservoir has a volume $V = 3.1\text{km}^3$, a surface area $A = 84\text{km}^2$ and a maximum depth $D = 190\text{m}$. The centre of mass of the water in the dam is a height $h_1 = 990\text{m}$ above the fjord below. The density of water is 1000kgm^{-3} to two significant figures.

a) How much gravitational potential energy does the reservoir contain relative to the water in the fjord?

Solution: To calculate the energy it is sufficent to consider as if all the mass of the reservoir was located at the centre of mass. So the gravitational potential energy is $mgh = \rho V gh = 1000 * 3.1 * 1000^3 * 9.8 * 990 = 3 \times 10^{16}$**J.**

b) If water is extracted from the dam at a rate $R = 37\text{m}^3\text{s}^{-1}$, and drops a height $h_2 = 920\text{m}$ through a generator which converts 85% of the gravitational potential energy into electricity, what is the power output of the generator?

Solution: This is just the rate at which gravitational potential energy is lost, times the efficiency $0.85 * 37 * 1000 * 9.8 * 920 = 280$**MW.**

c) If an electric car needs a force of $F = 550\text{N}$ to move along level ground at a constant speed $v = 70\text{km/h}$, how many electric cars would use an equivalent amount of electrical energy as supplied by the generator?

Solution: The car's motion has a power transfer of $P = Fv = 550 * 70 * 1000/3600 = 10,769$**W. So the hydroelectric generator can power** $280,000,000/10,769 = 26,384 = 26,000$ **cars to 2 significant figures. This ignores other possible losses in the car (e.g. air conditioning or heated seats).**

d) If the force on the cars scales with v^2 and the speed of the cars is increased from $v_1 = 70\text{km/h}$ to $v_2 = 80\text{km/h}$, how many fewer cars could be powered by the equivalent amount of energy supplied by the generator?

Solution: If the force scales as v^2**, then the power used by the car will scale like** v^3 **since** $P = Fv$**. So the new number of cars that can be supplied with the same power will be** $N_{new} = N_{old}\left(\frac{v_{old}}{v_{new}}\right)^3 = 26384 * \left(\frac{70}{80}\right)^3 = 17675$ **cars. That is 9000 cars fewer to 2 significant figures.**

FYS100 Physics – Formula sheet

Rotational motion about a fixed axis	**Translational motion**
Angular velocity $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Translational velocity $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Angular acceleration $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Translational acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Net torque $\sum_k \tau_k = I\,\alpha$	Net force $\sum_k F_k = m\,a$
$\alpha = \text{constant} \begin{cases} \omega_f = \omega_i + \alpha\,t \\ \theta_f = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\,t \end{cases}$	$a = \text{constant} \begin{cases} v_f = v_i + a\,t \\ x_f = x_i + v_i\,t + \frac{1}{2}a\,t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\,t \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Work $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Rotational kinetic energy $K = \frac{1}{2}\,I\,\omega^2$	Kinetic energy $K = \frac{1}{2}\,m\,v^2$
Power $\mathcal{P} = \tau\,\omega$	Power $\mathcal{P} = F\,v$
Angular momentum $L = I\,\omega$	Linear momentum $p = m\,v$
Net torque $\sum_k \tau_k = \frac{\mathrm{d}L}{\mathrm{d}t}$	Net force $\sum_k F_k = \frac{\mathrm{d}p}{\mathrm{d}t}$

General formulas	
Motion with constant acceleration	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\,t + \frac{1}{2}\,\vec{\mathbf{a}}\,t^2 \end{cases}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = m\,\vec{\mathbf{a}}$
Work	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Work-kinetic energy theorem	$\Delta K = W$
Linear momentum	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$
Impulse	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impulse-momentum theorem	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Center of mass	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \int \vec{\mathbf{r}}\,\mathrm{d}m$
Moment of inertia	$I = \int r^2\,\mathrm{d}m$
Parallel-axis theorem	$I = I_{\text{CM}} + M\,D^2$
Torque	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Angular momentum	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Net torque	$\sum_k \vec{\boldsymbol{\tau}}_k = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$
Rotational motion	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonic oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Mathematical rules

Vector relations	
Scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \cos\phi$
Magnitude of vector product	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \sin\phi$

Trigonometry	
Definitions	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identities	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Derivatives	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

Quadratic equations	
Equation	$a\,t^2 + b\,t + c = 0$
Solution	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Equation of a straight line	
Two points on the line are given	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$

Oppgave 1: Flytte en blokk

En mann med masse $m_1 = 84$kg ønsker å flytte en rektangulær blokk med masse $M = 110$kg på en flat overflate. Koeffisienten for statisk friksjon mellom blokken og overflaten er $\mu_s = 0,80$. Koeffisienten for kinetisk friksjon mellom blokken og overflatene er $\mu_k = 0,72$.

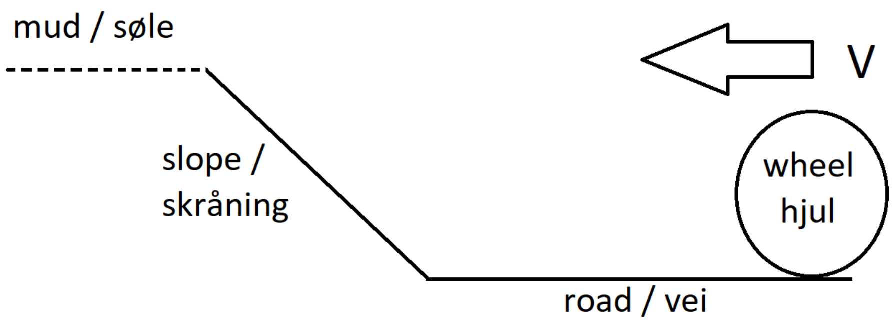
a) Opprinnelig sitter et barn med masse $m_2 = 27$kg på toppen av blokken. Hvis mannen skyver horisontalt på blokken med en kraft på $F_h = 990$N, vis om blokken vil bevege seg eller ikke.

b) Hva er verdien av friksjonskraften på blokken når mannen skyver på den?

c) Hvis barnet nå hopper av blokken, hva blir akselerasjonen av blokken når mannen skyver på den?

d) Hvis friksjonskoeffisientene mellom mannens føtter og overflaten er de samme som mellom blokken og overflaten, glir mannen når han skyver blokken? Er mannen i stand til å flytte blokken mens han glir og påfører en horisontal kraft på 990N på blokken?

Oppgave 2: Et rullende hjul



Et hjul med masse $m = 25$kg og radius $R = 0,27$m ruller langs en vei uten å gli.

a) Hvis hjulet beveger seg med en hastighet på 89km/t, hva er hjulets vinkelhastighet?

b) Det tar $0,69$ sekunder før hjulet gjør ti fullstendige omdreininger mens det ruller langs veien. Hvilken konstant vinkelakselerasjon ville hjulet ha hvis starthastigheten er 89km/t?

c) Ved enden av veien er det en skråning med konstant vinkel θ til den horisontale og totale lengden langs skråningen L. Vis at for at hjulet skal nå toppen av skråningen uten å gli, må translasjonshastigheten til hjulet på bunnen av skråningen være minst v_{min}, hvor

$$v_{\text{min}} = R\sqrt{\frac{2mgL\sin\theta}{(I + mR^2)}}, \tag{7}$$

g er akselerasjonen på grunn av tyngdekraften, og I er treghetsmomentet på hjulet. Oppgi antagelser eller tilnærminger du gjør for å utlede dette resultatet.

d) Hjulet kan modelleres som en solid sylinder med treghetsmoment rundt sin sentrale akse på $I = \frac{1}{2}mR^2$. På toppen av skråningen er det et veldig gjørmete felt, hvor rullemotstand bruker et konstant dreiemoment på hjulet på $\tau =$ 19Nm. Hvis hjulet stopper i $d = 2,9$m uten å skli, beregne hva den opprinnelige translasjonshastigheten var på toppen av skråningen før det gjørmete feltet.

Oppgave 3: Planetkollisjon

En solid sfærisk planet har en jevn tetthet på $\rho = 5500$kgm^{-3}, en radius på $R = 2300$km og en rotasjonsperiode på $T = 11$ timer.

a) Hva er størrelsen på planetens vinkelhastighet rundt dens rotasjonsakse?

b) Hvis man ser langs planetens rotasjonsakse i retning av vinkelhastigheten, roterer den med klokken eller mot klokken?

c) Ved bruk av treghetsmomentet til en solid kule på $I = \frac{2}{5}MR^2$, beregne størrelsen på planetens drivmoment.

d) En stor asteroide kolliderer nå med planeten. I et x, y, z koordinatsystem (med z retningen til planetens drivmoment beregnet ovenfor) har asteroiden totale vinkelmomentkomponenter på $(8,9 \times 10^{30}, 3,8 \times 10^{30}, -2.3 \times 10^{31})$ i enheter på kg m^{2}s^{-1}. Forutsatt at asteroiden er fullstendig absorbert av planeten i en perfekt uelastisk kollisjon, hva er størrelsen på det totale drivmomentet på planeten pluss asteroiden etter kollisjonen?

Oppgave 4: Vannkraft og biler

Et vannkraftreservoar har et volum på $V = 3.1\text{km}^3$, et overflateareal $A = 84\text{km}^2$ og en maksimal dybde $D = 190\text{m}$. Vannets massesenter i demningen har en høyde $h_1 = 990\text{m}$ over fjorden nedenfor. Tettheten av vann er 1000kgm^{-3} til to gjeldende sifre.

a) Hvor mye gravitasjonelle potensielle energi inneholder reservoaret i forhold til vannet i fjorden?

b) Hvis vann blir ekstrahert fra demningen med en strømningshastighet på $R = 37\text{m}^3\text{s}^{-1}$, og faller en høyde $h_2 = 920$ m gjennom en generator som konverterer 85% av dens gravitasjonelle potensielle energi til elektrisitet, hva er generatorens effekt?

c) Hvis en elektrisk bil trenger en kraft på $F = 550\text{N}$ for å bevege seg langs plan bakke med konstant hastighet $v = 70\text{km/t}$, hvor mange elektriske biler vil bruke en tilsvarende mengde elektrisk energi som generatoren genererer?

d) Hvis kraften på bilene skalerer med v^2 og hastigheten på bilene økes fra $v_1 = 70\text{km/t}$ til $v_2 = 80\text{km/t}$, hvor mange færre biler kan være drevet av den tilsvarende mengde energien levert av generatoren?

FYS100 Fysikk – formelark

Rotasjon om en fast akse	**Éndimensjonal bevegelse**
Vinkelhastighet $\omega = \frac{d\theta}{dt}$	Hastighet $v = \frac{dx}{dt}$
Vinkelakselerasjon $\alpha = \frac{d\omega}{dt}$	Akselerasjon $a = \frac{dv}{dt}$
Resultantmoment $I\,\alpha = \sum_k \tau_k$	Resultantkraft $m\,a = \sum_k F_k$
$\alpha = \text{konstant} \begin{cases} \omega_f = \omega_i + \alpha\, t \\ \theta_f = \theta_i + \omega_i\, t + \frac{1}{2}\alpha\, t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\, t \end{cases}$	$a = \text{konstant} \begin{cases} v_f = v_i + a\, t \\ x_f = x_i + v_i\, t + \frac{1}{2} a\, t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\, t \end{cases}$
Arbeid $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Arbeid $W = \int_{x_i}^{x_f} F\, dx$
Kinetisk energi $K = \frac{1}{2}\, I\,\omega^2$	Kinetisk energi $K = \frac{1}{2}\, m\, v^2$
Effekt $\mathcal{P} = \tau\,\omega$	Effekt $\mathcal{P} = F\, v$
Spinn $L = I\,\omega$	Bevegelsesmengde $p = m\, v$
Spinnsatsen $\frac{dL}{dt} = \sum_k \tau_k$	Newtons 2. lov $\frac{dp}{dt} = \sum_k F_k$

Generelle sammenhenger	
Bevegelse med konstant akselerasjon	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\, t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\, t + \frac{1}{2}\, \vec{\mathbf{a}}\, t^2 \end{cases}$
Newtons 2. lov	$m\, \vec{\mathbf{a}} = \sum_k \vec{\mathbf{F}}_k$
Arbeid	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$
Arbeid-kinetisk energi teoremet	$\Delta K = W$
Bevegelsesmengde	$\vec{\mathbf{p}} = m\, \vec{\mathbf{v}}$
Newtons 2. lov	$\frac{d\vec{\mathbf{p}}}{dt} = \sum_k \vec{\mathbf{F}}_k$
Impuls	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\, dt$
Impuls-bevegelsesmengde teoremet	$\Delta \vec{\mathbf{p}} = \vec{\mathbf{I}}$
Massesenter	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \int \vec{\mathbf{r}}\, dm$
Treghetsmoment	$I = \int r^2\, dm$
Steiners sats (parallellakseteoremet)	$I = I_{\text{CM}} + M\, D^2$
Kraftmoment	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Spinn	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Spinnsatsen	$\frac{d\vec{\mathbf{L}}}{dt} = \sum_k \vec{\boldsymbol{\tau}}_k$
Sirkelbevegelse	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonisk oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Matematiske sammenhenger

Vektorrelasjoner	
Prikkprodukt	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \|\vec{\mathbf{A}}\|\,\|\vec{\mathbf{B}}\|\,\cos\phi$
Absoluttverdi av kryssprodukt	$\|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\| = \|\vec{\mathbf{A}}\|\,\|\vec{\mathbf{B}}\|\,\sin\phi$

Trigonometri	
Definisjoner	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identiteter	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Deriverte	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

2. grads ligning	
Ligning	$a\,t^2 + b\,t + c = 0$
Løsning	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Ligningen for en rett linje	
Gitt to punkter på linjen	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$