3 Assignment

In this assignment, which comprises only mandatory exercises, you will practice with using the Laplace transform and the transfer function in order to compute the output response of an LTI system.

You will find the following Laplace transforms useful:

$$\mathcal{L}\{e^{at}1(t)\} = \frac{1}{s-a} \qquad \mathcal{L}\{\sin(\omega t)1(t)\} = \frac{\omega}{s^2+\omega^2} \qquad \mathcal{L}\{\cos(\omega t)1(t)\} = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\{1(t)\} = \frac{1}{s} \qquad \mathcal{L}\{te^{at}1(t)\} = \frac{1}{(s-a)^2} \qquad \mathcal{L}\{\frac{d}{dt}f(t)\} = sF(s) - f(0)$$

3.1 Laplace transform (mandatory)

Question 3.1

Given the following differential equation:

$$\dot{y}(t) = -3y(t) + u(t) \tag{3.1}$$

with initial value $y(0) = 5$. Assume that the input variable $u(t)$ is a step signal of amplitude 2 at time $t = 0$.

1. Compute the corresponding output response $y(t)$ using the Laplace transform.
2. Calculate the steady-state value of $y(t)$ (the final value of $y(t)$ when $t \to \infty$) using the Final Value Theorem. Then, calculate the steady-state value y_{ss} using the $y(t)$ computed at point 1. Are the two computed values the same?
3. According to the time-derivative property of the Laplace transform, $\mathcal{L}\{\dot{y}(t)\} = sY(s) - y_0$. Under the assumption that $y_0 = 0$, we obtain the following from (3.1):

$$sY(s) = -3Y(s) + U(s) \qquad \Rightarrow \qquad Y(s) = H(s)U(s) = \frac{1}{s+3}U(s) \tag{3.2}$$

   where $H(s)$ is the *transfer function*. Using MATLAB functions `tf` and `step`, write a code that plots the step response of (3.2). Then, using the Simulink blocks `Step`, `Transfer Fcn` and `Scope`, perform a simulation of the step response. Are the obtained signals $y(t)$ the same in both cases? Compare them with the expression of $y(t)$ that you obtained at point 1 of this exercise, and discuss similarities/differences.

3.2 Transfer function (mandatory)

Question 3.2

Consider the mass-spring-damper system in Fig. 3.5, where y denotes the position, F is the applied force, D is the damping coefficient, K is the spring constant. By assuming that the damping force F_d is proportional to the velocity, and that the spring force F_s is proportional to the position of the mass, and such that $F_s = 0$ when $y = 0$, then the following equation is obtained from the force balance:

$$m\ddot{y}(t) = F(t) - D\dot{y}(t) - Ky(t) \tag{3.3}$$

Calculate the transfer function from the force F to the position y.

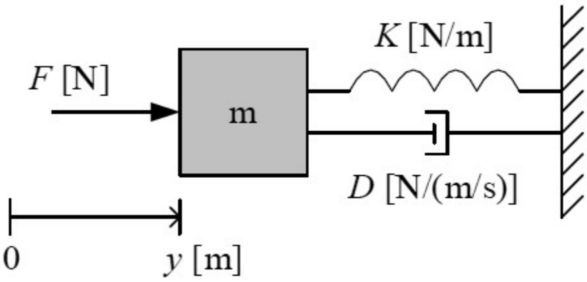

Figure 3.1: Mass-spring-damper system.

3.3 Output response using the transfer function (mandatory)

Question 3.3

1. Consider an LTI system described by the transfer function $H(s) = \frac{1}{(s+2)}$. Compute the output response $y_1(t)$ which corresponds to an input signal $u_1(t) = \sin(\omega t)1(t)$.
2. Imagine that the transfer function had been $H(s) = \frac{1}{(s+4)}$ instead. Which term in $y_1(t)$ could you change instantaneously without performing any calculation, and how? *Note: Of course, in order to get the correct entire expression for $y_1(t)$ you would still need to perform again all the calculations, you can do this as an optional exercise for familiarizing yourself further with the involved calculations.*
3. Consider an LTI system described by the transfer function $H(s) = \frac{s-1}{(s+1)^2}$. Compute the output response $y_2(t)$ which corresponds to the input signal $u_2(t) = 1(t)$.