4 Assignment

In the mandatory exercises of this assignment, you will practice with the relationship between the state-space and the transfer function. You will also get a deeper understanding of BIBO stability and internal stability. Finally, you will practice with the simplification of a block diagram.

4.1 From state-space to transfer functions (mandatory)

Question 4.1

Obtain the transfer function of the system defined by:

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + x_2(t) \\ \dot{x}_2(t) = -x_2(t) + x_3(t) \\ \dot{x}_3(t) = -2x_3(t) + u(t) \\ y(t) = x_1(t) \end{cases} \tag{4.1}$$

Discuss about the external (BIBO) stability and internal stability of this system.

Question 4.2

Obtain the transfer function of the system defined by:

$$\begin{cases} \dot{x}_1(t) = -39x_1(t) + 84x_2(t) + 7u(t) \\ \dot{x}_2(t) = -18x_1(t) + 39x_2(t) + 3u(t) \\ y(t) = x_1(t) - 2x_2(t) \end{cases} \tag{4.2}$$

Discuss about the external (BIBO) stability and internal stability of this system.

4.2 Asymptotic and BIBO stability (mandatory)

Question 4.3

Consider the system given by the following equations:

$$\begin{cases} \dot{x}_1(t) = x_2(t) - x_2(t)^2 x_1(t) \\ \dot{x}_2(t) = -5x_1(t) - 2x_2(t) \end{cases} \tag{4.3}$$

(a) Linearize the system about the equilibrium point $(\bar{x}_1, \bar{x}_2) = (0, 0)$ and discuss the internal stability of the linearized system.

(b) Consider a system described by the differential equation $\ddot{x}(t) + p\dot{x}(t) + qx(t) = u(t)$. For which values of the parameters p and q is the system BIBO stable?

(c) Plot the impulse response of the system for $p = 2$ and $q = 5$ (you can use MATLAB functions `tf` and `impulse` to do this).

(d) Now let $p = 2$ and $q = -5$. Is the system BIBO stable? Plot the impulse response.

4.3 Block diagrams (mandatory)

Question 4.4

Simplify the block diagram shown in the figure, so that you obtain the equivalent transfer function $H_{RY}(s)$ from $R(s)$ to $Y(s)$. Using Simulink, show that the responses obtained using the original block diagram and the simplified block diagrams are equivalent (*you can use any type of input signal to show the equivalence*). What are the static gain $H_{RY}(0)$, the zeros and the poles of $H_{RY}(s)$? What type of exponential/trigonometric functions would we find in the step response of $H_{RY}(s)$? Is $H_{RY}(s)$ a BIBO stable system? What can we say about the internal stability of the underlying state-space representation?

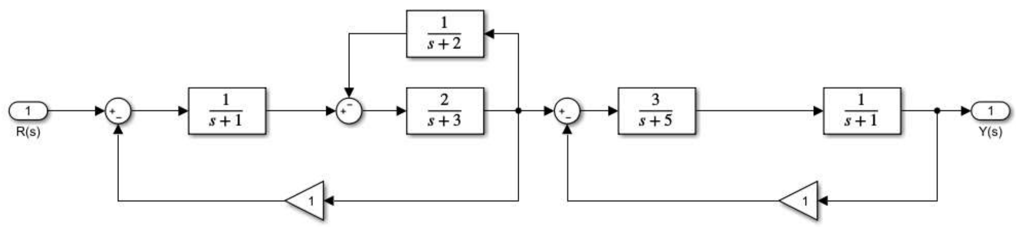

Figure 4.1: Block diagram of the system.

4.4 Generic time response (optional)

In this exercise, we are going to use the file *HW4generictime.exe*, which contains a ready-to-use simulator developed by UNED. The app is in Spanish, so you are referred to Fig. 4.5 for an English translation.

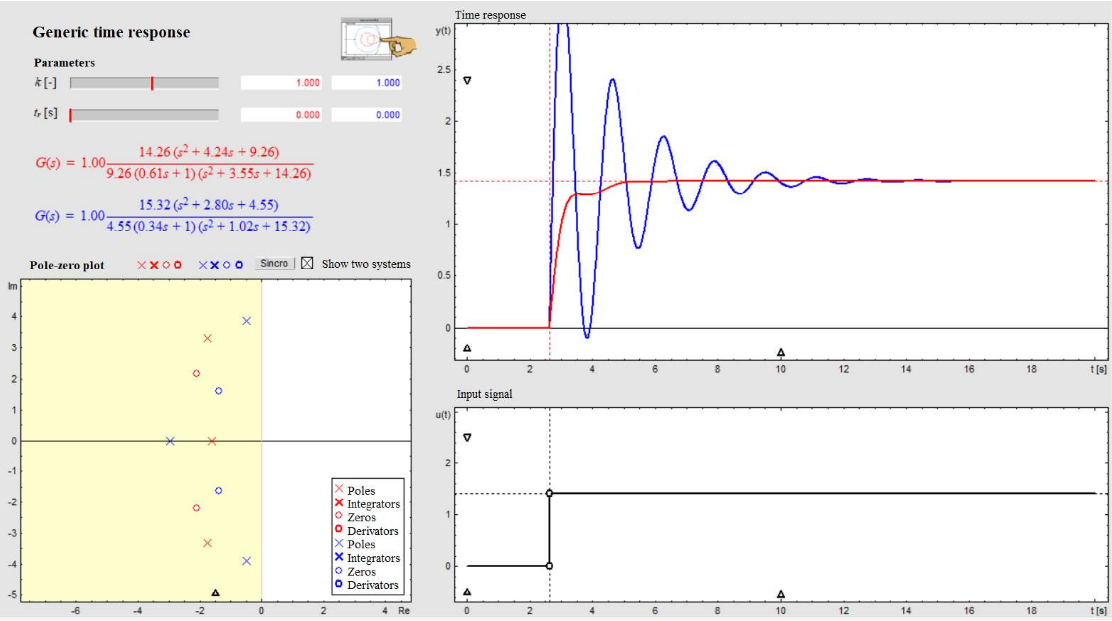

Figure 4.2: Software generic time response.

The goal of this exercise is to understand the effect of the static gain, the poles (roots of the denominator of $H(s)$) and the zeros (roots of the numerator of $H(s)$) on the time response. The simulator requires that the transfer functions are proper. This means that the degree of the denominator must be always bigger than or equal to the degree of the numerator. In other words, the transfer function must have a number of poles bigger than or equal to the number of zeros.

The simulator is divided into four parts:

- **Parameters:** In the upper-left part of the simulator you can modify the *static gain* k (this means $H(0)$) and the *time delay*[1] t_r.
- **Pole-zero plot:** this simulator includes a pole-zero editor, which can be used to modify the transfer function under consideration. You can modify the system by dragging poles and zeros inside the plot (the bold symbols represent integrators and derivators, i.e.,

[1] Some systems do not react instantaneously to changes in the input signal. This delay can be represented by a term e^{-st_r} appearing in the transfer function. You can learn more about the analysis and control of delayed systems in ELE600 - Videregående reguleringsteknikk at Master level. In ELE320, we assume that $t_r = 0\,s$, so you can set this parameter to 0.

poles and zeros in the origin). Note that all time the system must satisfy the following requirements:

  - maximum order 4
  - number of poles ≥ number of zeros

- **Time response:** this plot shows the step response of the system.
- **Input signal:** this plot shows the step input. You can modify both the magnitude and the step time (time at which $u(t)$ passes from 0 to 1).

Question 4.5

Use this software to gain more insight into how the position of poles and zeros affects the time behavior of a linear system (also keep an eye on how the transfer function changes). What happens when a zero is close to a pole? What happens when there are poles or zeros in the right half of the complex plane? What is the effect of the parameter k on the system's response?