5 Assignment

In this assignment, you are first going to work with two mandatory assignments that will help you in understanding the theoretical behavior of first-order and second-order systems. Afterwards, another mandatory exercise will make you work with a simulated real-world scenario concerning a first-order system available in our kitchens (a hob, på norks kokeplate). Finally, you are advised to take a look at the three optional exercises at the end of this document, which would strengthen your understanding of the behavior of first-order, second-order (without zeros) and second-order (with a real zero) systems.

5.1 First-order systems (mandatory)

Question 5.1

Given the following transfer functions:

$$H_1(s) = \frac{1}{4s+1} \tag{5.1}$$

$$H_2(s) = \frac{2}{4s+5} \tag{5.2}$$

(a) Find the poles and the 2% and 5% settling times for these transfer functions.

(b) Let the input signal $u(t)$ be a unit step. Find the corresponding step responses $y_1(t)$ and $y_2(t)$. *Hint: Write $H_1(s)$ and $H_2(s)$ in the standard form $H(s) = H(0)/(1 + s\tau)$.*

(c) Find the steady-state value of $y_1(t)$ and $y_2(t)$.

(d) Draw the step responses $y_1(t)$ and $y_2(t)$.

5.2 Second-order systems (mandatory)

Question 5.2

For each of the following transfer functions:

$$H_1(s) = \frac{15}{s^2 + 1.6s + 16} \tag{5.3}$$

$$H_2(s) = \frac{5}{2s^2 + 8s + 8} \tag{5.4}$$

$$H_3(s) = \frac{10}{s^2 + 7s + 10} \tag{5.5}$$

(a) Find the poles of the transfer function.

(b) Find the system's static gain $H(0)$, the natural frequency ω_0 and the damping ratio ζ. Determine the type of system (*undamped*, *underdamped*, *critically damped* or *overdamped*).

(c) For the underdamped systems from point (b), find the maximum overshoot, the 5% settling time, the 2% settling time, the peak time and the rise time.

(d) Using the final value theorem, determine the steady-state value when $u(t) = 1(t)$.

(e) Using MATLAB, obtain the responses of $y_1(t)$, $y_2(t)$ and $y_3(t)$ to the unit step $u(t) = 1(t)$, and check that they correspond to the values obtained at point (d). Moreover, in the case of underdamped systems, also check that the obtained responses correspond to the values computed at point (c).

5.3 Hob (NO: kokeplate) (mandatory)

Let's consider a hob (på norsk: kokeplate), as those you can find in your home. The following equation describes the typical dynamics of the temperature $T(t)$:

$$\dot{T}(t) = \frac{P(t)}{mc_p} - \frac{hA}{mc_p}\left(T(t) - T_{amb}(t)\right) \tag{5.6}$$

where:

- $P(t)$ [W] is the electrical power applied to the hob;
- $T(t)$ [°C] is the temperature of the hob, assumed to be evenly distributed;
- $T_{amb}(t) = 20$°C is the ambient temperature in the kitchen;
- $m = 0.5$ kg is the mass;
- $c_p = 460$ J/(kg°C) is the specific heat capacity of the hob;

- $h = 20\,\mathrm{J/(sm^2\,°C)}$ is the specific heat transfer coefficient between the hob and the air;
- $A = 0.1\,\mathrm{m}^2$ is the area of the hob/air surface.

In the following, we will consider that:

- $T(t)$ is the output variable $y(t) = T(t)$;
- $P(t)$ is the input variable $u(t) = P(t)$;
- $T_{amb}(t)$ is the disturbance signal $w(t) = T_{amb}(t)$.

Question 5.3

Do the following:

(a) Laplace transform (5.6) and find the transfer function between $U(s)$ and $Y(s)$ as:

$$H_p(s) = \frac{Y(s)}{U(s)} = \frac{H_p(0)}{1 + s\tau} \tag{5.7}$$

What are the values of $H_p(0)$ and τ?

(b) Find the transfer function from $W(s)$ to $Y(s)$ as:

$$H_w(s) = \frac{Y(s)}{W(s)} = \frac{H_w(0)}{1 + s\tau_w} \tag{5.8}$$

What are the values of $H_w(0)$ and τ_w? Explain why $H_w(0) = 1$.

(c) Find the poles of $H_p(s)$. Confirm the result by using the `pzmap` command in MATLAB.

To simulate the hob, we will use the Simulink model `kokeplate_tidskonstant.slx` which is shown in Fig. 5.1

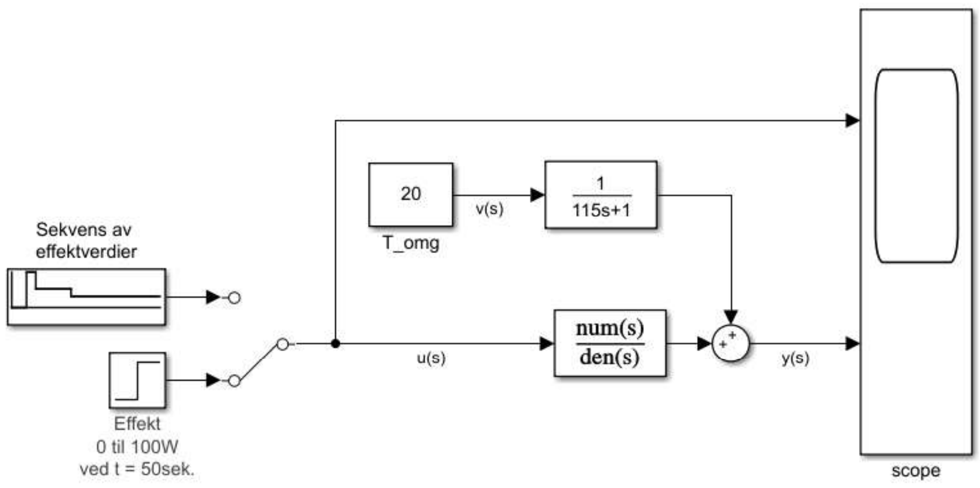

Figure 5.1: Simulink model of the hob.

To avoid an initial transient response from $0°C$ to $20°C$ due to the ambient temperature (before turning on the power), Simulink has a block called `Transfer Fcn (with initial output)` which you can find in the Simulink Extras folder under Additional Linear. Therein, we specify the system to have an initial output of 20 (same as the constant block which represents $T_{omg}(t)$). Double-click on the block to see how this is specified.

The model uses the following simulation configuration parameters:

- step time: 500 s
- integration method: Euler
- fixed step length: 1 s

By simulating the model, you apply 100 W at $t = 50$ s (this corresponds to set 1 on the switch). Remember that the total response consists of the sum of the partial responses due to the ambient temperature and the input:

$$Y(s) = H_p(s)U(s) + H_w(s)W(s) \tag{5.9}$$

Question 5.4

Based on the response in Simulink, determine the static gain $H_p(0)$ and the time constant τ for the transfer function $H_p(s)$. You can do this using the built-in Cursor Measurement function in Simulink, see Fig. 5.2.

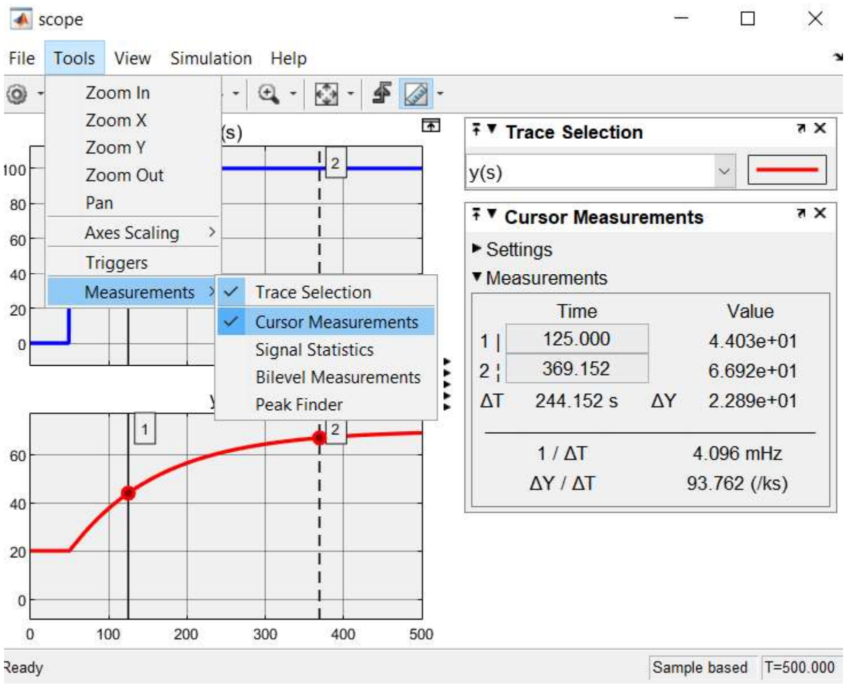

Figure 5.2: Tool for reading response data.

Question 5.5

Let us assume that you think that the time constant was relatively long when you turned the hob to 1, and that you would rather examine the time constant when you set the hob to 2, which is equivalent to 200 W. Simulate the model with this new input and determine the gain and time constant. Have they changed, and if so, to what value? What has this taught you?

As a chef, how would you get the hob to achieve a temperature of 120°C as soon as possible (without overshoot)? The hob can be turned up to power 6, i.e., 600 W. To find out this, double-click on the Switch so that you use the block `Sekvens av effektverdier` (this is a Source block called Repeating Sequence Stair). By double-clicking on the block you get the window shown in Fig. 5.3.

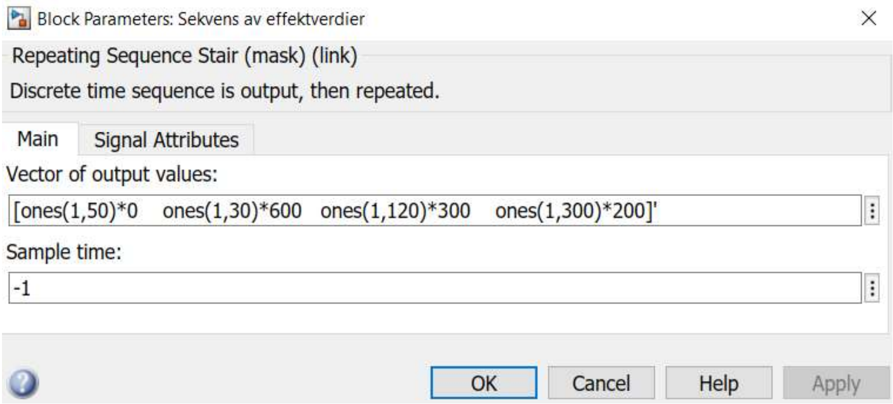

Figure 5.3: Parameterization of the hob input.

In Fig. 5.3, 0 W is used for the first 50 seconds, followed by 600 W for 30 seconds, then 300 W for 120 seconds and finally 200 W for 300 seconds. Make sure that the sum of the lengths of the `ones` is the same as the simulation time (500), e.g., 50+30+120+300 = 500. Always use 0 W in the first 50 seconds so that the first step input happens at 50 seconds.

Question 5.6

Your task is to find the sequence of input values that makes the time required by the hob to reach 120°C as short as possible (remember that the temperature should not exceed 120°C either). In this way, the response time T_r (i.e., the time required to reach 63% of the final value using active manipulation of the control signal) will be as short as possible, and you would act like a human temperature controller for the hob. What value do you find for the response time T_r when you choose actively the input signal? How much smaller is it when compared to the open-loop time constant?

Comment: As you have now learned, the open-loop time constant tells you how the system reacts to a step input of arbitrary magnitude. However, applying a step input is not the most efficient way to get a system to swing to a new value of its output: it is possible to make a system to behave faster by using actively the input signal, which is exactly what the controller will do *automatically* in a feedback control system.

5.4 First-order time response (optional)

In this exercise, we are going to use the file *HW5firstorder.exe*, which contains a ready-to-use simulator developed by UNED. The app is in Spanish, so you are referred to Fig. 5.4 for an English translation.

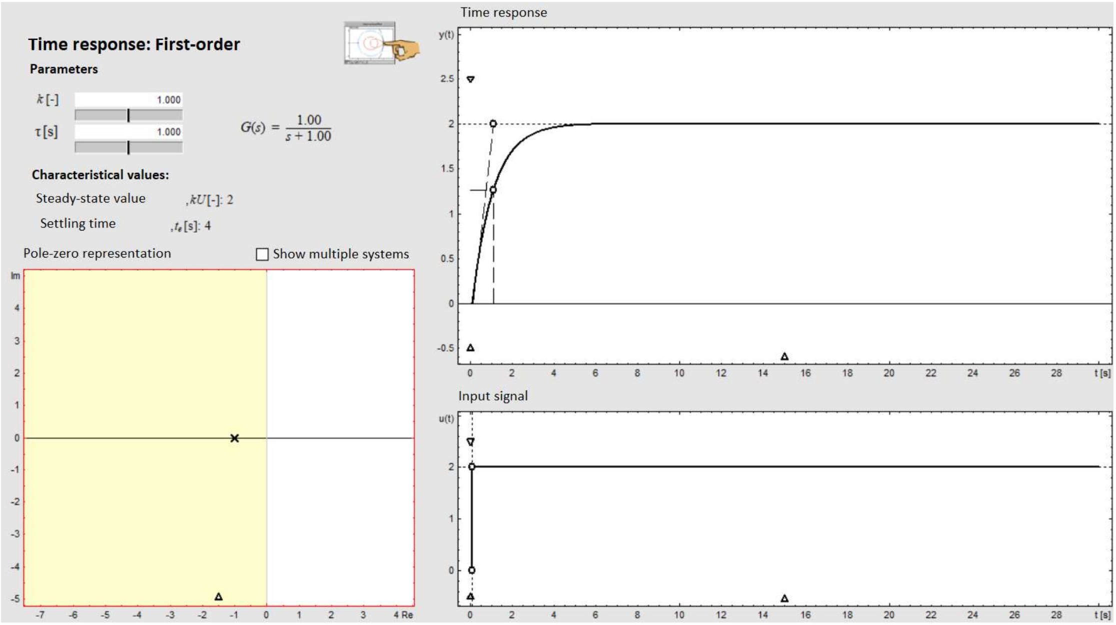

Figure 5.4: Software first-order response.

This simulator allows analyzing the step response of a first-order LTI system. The simulator is divided into four parts, and is completely interactive: any modification in one of the four parts will modify the remaining parts accordingly.

In the upper-left part of the simulator you will see the parameters (which can be modified): the static gain $k = G(0)$, together with the steady-state output value and the 2% settling time (4τ).

In the lower-left part of the simulator you will find the pole-zero representation: by dragging the pole (cross) left or right, you can modify its position in the complex plane, and observe the effect on the parameters, characteristics and output response of the system.

In the upper-right part of the simulator you will see the output response. You will see two circles: the upper one can be used to modify the final value of the response, whereas the lower one can be used to modify τ (this circle corresponds to 63% of the final value of the output, hence the value projected on the time axis is τ!)

In the lower-right part of the simulator you will see the input signal. You can modify this signal using the two circles: the upper one will modify the step amplitude, whereas the second circle can be used to modify the time at which the step signal switches from 0 to its final value.

Finally, by enabling *Show multiple systems*, five different systems will be shown so that you can perform a simultaneous comparison of how they behave. In the Menu *Opciones* (*Options*), you can select *Efecto ganancia* (i.e., *gain effect*) to generate five systems with different values of the static gain $k = G(0)$ but the same value of $\tau = 1$ s). Alternatively, you can select *Efecto constante tiempo* (*time constant effect*) to generate five systems with different values of τ but the same value of $k = G(0) = 1$.

Question 5.7

Use this software to gain more insight into how first-order systems behave. You master them when you are able to predict the effect of any modification applied in the simulator.

5.5 Second-order time response (optional)

In this exercise, we are going to use the file *HW5secondorder.exe*, which contains a ready-to-use simulator developed by UNED. The app is in Spanish, so you are referred to Fig. 5.5 for an English translation.

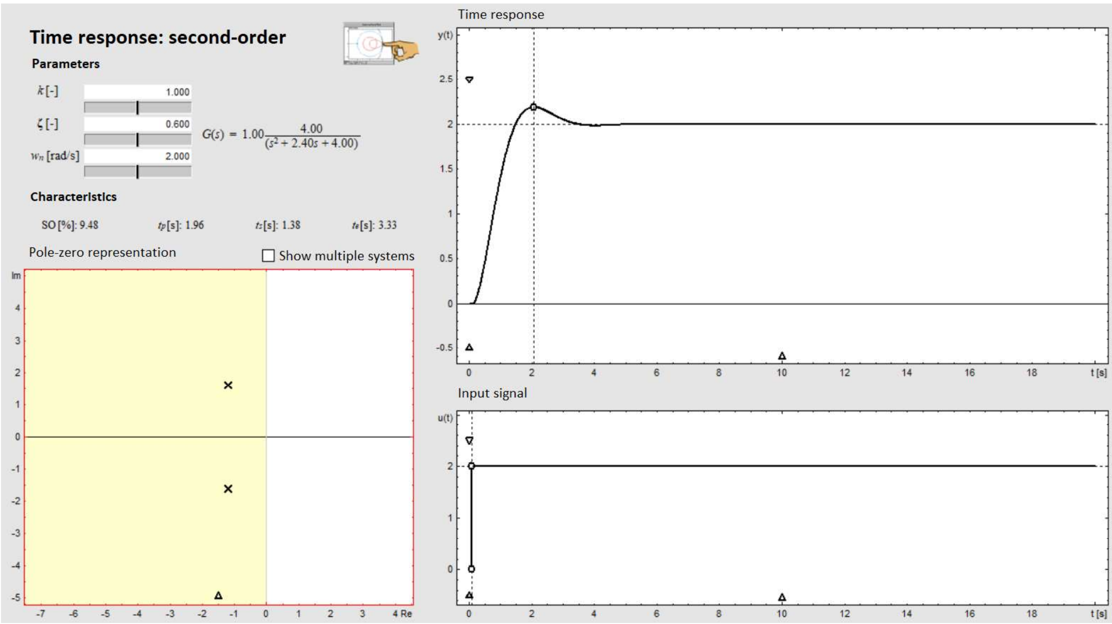

Figure 5.5: Software second-order response.

The structure of this app is similar to the previous one. In the upper-left part, you can modify the following parameters: the static gain k, the damping ratio ζ, and the natural frequency w_n. In case a value $\zeta \geq 1$ is introduced, then the parameters will change to τ_1 and τ_2, which describe the time constants of an overdamped (or critically damped) system.

Below the parameters, there appear the characteristics of the system[1]:

- the *maximum overshoot*, which expresses the difference between the magnitude of the highest peak of the time response and the magnitude of its steady-state:

$$SO[\%] = 100\exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

- the *peak time*, which is the time required for the response to reach the first peak of the overshoot:

$$t_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}}$$

- the *rise time*, which is the time required to go from 0% to 100% of its final value:

$$t_s = \frac{1}{w_n\sqrt{1-\zeta^2}}\left[\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

- the *2% settling time*, which is the time required by the response to reach and be permanently within the specified 2% range of the final value:

$$t_e \approxeq \frac{4}{\zeta w_n}$$

  Note that in the overdamped/critically damped case, its value is given by:

$$t_e \approxeq 4\left(\tau_1 + \tau_2\right)$$

In the lower-left part of the simulator you will find the pole-zero representation: by dragging the poles, you can modify their position in the complex plane and observe the effect on the parameters, characteristics and output response of the system. Note that by dragging the poles close to the real axis, the system passes from underdamped to overdamped.

In the upper-right part of the simulator you will see the output response, which you can modify by either dragging the black circle or the dashed lines (note that any modification of the output response will lead to changes in some parameters' values and possibly in the poles' position in the complex plane).

In the lower-right part of the simulator you will see the input signal. You can modify this signal using the two circles: the upper one will modify the step amplitude, whereas the second circle can be used to modify the time at which the step signal switches from 0 to its final value.

Finally, by enabling *Show multiple systems*, five different systems will be shown so that you can perform a simultaneous comparison of how they behave. In the Menu *Opciones* (*Options*), you can select:

[1]The names of the parameters come from the Spanish *sobreoscilación*, *tiempo de pico*, *tiempo de subida* and *tiempo de establecimiento*.

- *Efecto factor amortiguamento* (*damping ratio effect*): you will compare systems with the same natural frequency w_n, but different damping ratios ζ. Note that this corresponds to moving the poles along a circle (you can drag the dashed circle to change the value of w_n).
- *Efecto frecuencia no amortiguada* (*undamped frequency effect*): you will compare systems with the same damping ratio ζ, but different natural frequencies w_n (this frequency is also called undamped frequency since it corresponds to the frequencies of oscillations that would be obtained with $\zeta = 0$). Note that this corresponds to moving the poles along a line of the complex plane (you can drag the dashed lines to change the slope, and thus the value of ζ).
- *Efecto parte imaginaria constante* (*constant imaginary part effect*): you will compare systems which have the same imaginary part of the poles. Note that in this case, each system will have a different pair of values ζ and w_n. By dragging the horizontal dashed line, you can change the value of the imaginary part of the poles.
- *Efecto parte real constante* (*constant real part effect*): the same as above, but with the real part kept constant.

Question 5.8

Use this software to gain more insight into how second-order systems behave. You master them when you are able to predict the effect of any modification applied in the simulator.

5.6 Second-order time response with zero (optional)

In this exercise, we are going to use the file *HW5secondorder0.exe*, which contains a ready-to-use simulator developed by UNED. The app is in Spanish, so you are referred to Fig. 5.6 for an English translation.

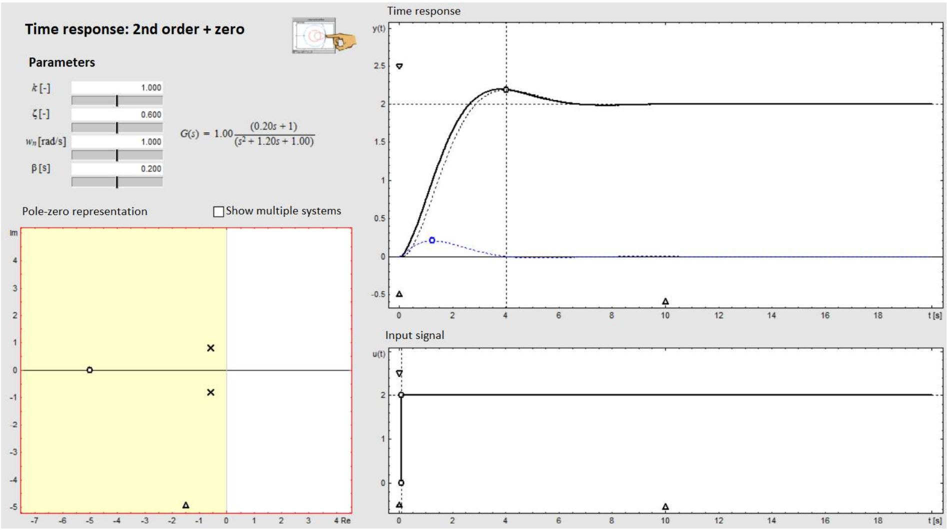

Figure 5.6: Software second-order response with zero.

In the upper-left part of the simulator you can modify the static gain k, the damping factor ζ, the natural frequency w_n and the parameter β, which corresponds to the real zero in $s = -1/\beta$. The resulting transfer function is given by:

$$G(s) = k\frac{w_n\,(s\beta + 1)}{s^2 + 2\zeta w_n s + w_n^2} \tag{5.10}$$

In the lower-left part of the simulator you can move the poles (crosses) and the zero (circle) by dragging them.

In the upper-right part of the simulator, you will see the output response:

- the dashed black line corresponds to the response without the zero
- the solid black line corresponds to the response with the zero
- the blue dashed line corresponds to the contribution of the zero to the system's response

In the lower-right part of the simulator, you will find the input signal, of which both the magnitude and the step time can be modified by dragging the corresponding black circles.

Finally, by enabling *Show multiple systems*, four different systems will appear corresponding to four positions of the zero, so that you can perform a simultaneous comparison of how they behave.