7 Assignment

7.1 Control of a third-order process

7.1.1 Introduction

In Canvas, you will find the Simulink file `reg_sys.slx` which will be used to simulate the feedback control system for the third-order process:

$$H(s) = \frac{0.1}{(15s+1)(10s+1)(5s+1)} = \frac{0.1}{750s^3 + 275s^2 + 30s + 1} \tag{7.1}$$

We will assume that the measuring instrument (situated on the feedback loop) has a time constant of 2 seconds and a static gain of 1, so that it is described by a transfer function:

$$M(s) = \frac{1}{1+2s} \tag{7.2}$$

A PID with filtered derivative effect has been implemented (red block). By double clicking on it, you can modify its parameters. You will see that the option of specifying the integrator limits is available, which you can do for example by replacing the values under `Max_paadrag/Min_paadrag` from `Inf/-Inf` to `100/-100`. This is done to avoid that the integrator keeps integrating the error signal when the actuators are already working at their maximum value, which is technically known as *anti-windup*. The reference signal goes from 0 to 1 at time t = 10 seconds, and then back to 0 at time t = 210 seconds.

7.1.2 Routh-Hurwitz criterion

Let us consider the following proportional controller transfer function, where K_p is the proportional gain[1]:

$$C(s) = K_p \tag{7.3}$$

[1] Here are a few cool commands you might want to try in MATLAB, although you will most likely not use them in this exercise: lorenz, image, penny, why, xpbombs.

Question 7.1

Use the Routh-Hurwitz criterion to determine the inequalities that must be satisfied by K_p in order to get BIBO stability of the closed-loop transfer function:

$$G(s) = \frac{H(s)C(s)}{1 + H(s)C(s)M(s)}$$

Then, simulate the system with a value of K_p slightly lower and slightly bigger than the upper bound that you computed. Do the simulations confirm the theoretical analysis?

Hint: If you compute $H_{cl}(s)$ correctly, then its denominator will have the form $a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4$. It might be convenient to divide every coefficient by a_0 to get a polynomial in the form $s^4 + b_1s^3 + b_2s^2 + b_3s + b_4$ before building the Routh-Hurwitz table. Moreover, something to remember to ease the calculations is that you can always multiply all the numbers in the same row of the table by the same positive number, without changing the final inference obtained from the table.

7.1.3 Manual tuning of the PID controller

You will now tune manually the controller (Table 7.1 resumes the effect of increasing each individual parameter).

↑ parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	Decrease	Increase	Small change	Decrease	Degrade
K_i	Decrease	Increase	Increase	Eliminate	Degrade
K_d	Minor change	Decrease	Decrease	No effect	Improve (if K_d small)

Table 7.1: Effect of increasing PID parameters.

For a PI controller, you should first increase the proportional gain K_p until satisfactory stability is seen in the response of $y(t)$ to the step change in $r(t)$. Then increase the integral gain K_i (which means decreasing the integral time T_i) until the closed-loop system is almost unstable. Finally, tune again K_p until the stability is deemed satisfactory.

Question 7.2

Let `Max_paadrag/Min_paadrag` be `Inf/-Inf` and find values for K_p and T_i for a PI controller. Remember that there is not a single correct solution, but you need to find some values that provide a response that you consider to be satisfactory. Try to get a fast response without too much overshoot. Include the parameter values and the obtained plots.

For a PID controller, increase the gain K_p until satisfactory stability is seen in the response of

$y(t)$ to the step change in $r(t)$. Then, activate the integral and derivative terms and tune them taking into account Table 7.1.

Question 7.3

Now, find a good value for the parameter T_d to be used in the PID controller. You will need to choose also the parameter T_f: a rule of thumbs is to choose is approximately as $T_f \approx 0.2T_d$. Compare the response obtained using the PID controller with the response obtained with the PI controller.

7.1.4 Control without anti-windup

In order for you to be able to see the effect of the integrator limitation, you must increase the reference change so that the actuator goes into saturation.

You must still let `Max_paadrag/Min_paadrag` to be `Inf/-Inf`. To illustrate the phenomenon, you should set $T_d = 0$, but keep the other parameters that you found. The reason for this is that the derivative effect makes the input very hectic, so that it becomes hard to interpret from the plots what is going on.

Question 7.4

Change `Ny_Verdi` in the reference block to 12, so that the reference goes from 0 to 12 at t = 0 and back to 0 at t = 210 s. Then simulate the model, and observe the input and output signals. Explain in words what is happening.

7.1.5 Control with anti-windup

Specify now `Max paadrag/Min paadrag` to 100/-100.

Question 7.5

Use the PID parameters from Question 7.3. Simulate again and observe the computed control input and the obtained output response. Describe what is happening.

7.2 Control of first-order processes

You will now design a controller for the following first-order system:

$$H(s) = \frac{0.5}{200s + 1} \tag{7.4}$$

7.2.1 P-controller

Let us start first with the proportional controller (K_p is the proportional gain):

$$C(s) = K_p \tag{7.5}$$

Question 7.6

Find out the closed-loop transfer function $H_{cl}(s) = Y(s)/R(s)$, where $Y(s)$ is the output of $H(s)$ and $R(s)$ is the reference signal, under the assumption that the input to the controller is $E(s) = R(s) - Y(s)$. Put $H_{cl}(s)$ in the standard first-order form and find the expression of how the pole of $H_{cl}(s)$ moves when K_p increases.

Question 7.7

From $H_{cl}(s)$, find an expression of how the closed-loop time constant τ_{cl} and the closed-loop static gain $H_{cl}(0)$ vary as a function of K_p.

Question 7.8

What value of K_p should you use to obtain $\tau_{cl} = 100$ s?

Question 7.9

Implement the feedback control system comprising process + controller (with the K_p computed as answer to Question 7.8). Use a unit step signal and simulate with a total length of 500 s. Use a fixed-step Euler integration method with fixed-step size 1 s (you can change this parameter by right-clicking on a blank point of the Simulink scheme, then select `Model configuration parameters`). Provide two plots: i) comparison between the reference $r(t)$ and the output $y(t)$; ii) usage of the control input $u(t)$. What are the closed-loop time constant and closed-loop static gain as read from the plots? Do they correspond to the theoretical ones?

Question 7.10

What is the steady-state error $e_\infty = \lim_{t\to\infty} e(t)$? Verify this value by applying the final value theorem either directly to $E(s)$ or indirectly to $Y(s)$, which means that after computing $y_\infty = \lim_{t\to\infty} y(t)$, you calculate $e_\infty = r_\infty - y_\infty$.

7.3 PI-controller

To eliminate the steady-state error to a step in the reference signal $R(s)$, you will use a PI controller given by:

$$C(s) = K_p \frac{1 + sT_i}{sT_i} \tag{7.6}$$

where K_p is the proportional gain and T_i is the integral time.

Question 7.11

Find the closed-loop transfer function $Y(s)/R(s)$.

Question 7.12

Prove analytically that the steady-state error to a step change in the reference signal $R(s)$ is $e_\infty = 0$ when you use a PI controller *(as long as the closed-loop system is BIBO stable).*

Question 7.13

Confirm the theoretical result from the previous question by simulating the feedback control system with the PI controller in Simulink. Choose values of K_p and T_i so that the closed-loop system is BIBO stable. Let the length of the simulation be 500 seconds.

Question 7.14

Assume that the closed-loop system must have a natural frequency $\omega_0 = 0.01$ rad/s and a damping factor $\xi = 0.75$. How should T_i and K_p be selected to satisfy these specifications? Use Simulink to simulate the closed-loop response (use a simulation length of 500 seconds), and discuss how the zero at $s = -1/T_i$ in $H_{cl}(s)$ affects the response.