LØSNINGSFORSLAG MARS -21

1) $z = 2-3i \quad , w = 3-i$

$$z^2 = (2-3i)^2 = 2^2 - 2\cdot 3i \cdot 2 + (3i)^2$$

$$= 4 - 12i + 9\cdot(-1) = \underline{\underline{-5-12i}}$$

$$\bar{z}w = (2+3i)(3-i) = 6 + 9i - 2i - 3i^2$$

$$= \underline{\underline{9+7i}}$$

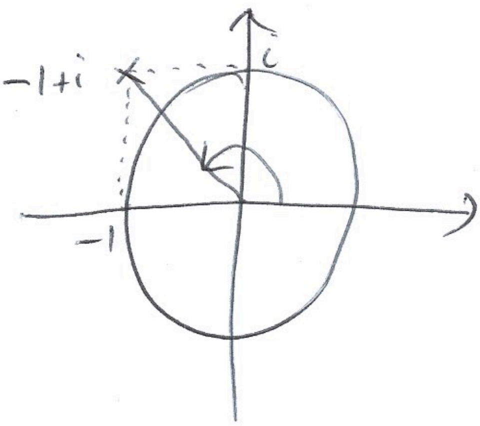
$$\frac{z}{w} = \frac{zw}{|w|^2} = \frac{(2-3i)(3-i)}{9+1}$$

$$= \frac{6 - 2i - 9i + 3i^2}{10} = \frac{3-11i}{10} = \underline{\underline{\frac{3}{10} - \frac{11}{10}i}}$$

b) $z^3 + 1 - i = 0$

$$z^3 = -1+i$$

$$z^3 = \sqrt{2}\, e^{\frac{3\pi}{4}i}$$



$$z_1 = (\sqrt{2})^{\frac{1}{3}} \cdot e^{\frac{3\pi}{4\cdot 3}i} = 2^{\frac{1}{6}} e^{\frac{\pi}{4}i}$$

$$= 2^{\frac{1}{6}} \cdot \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)$$

$$= \underline{\underline{2^{-\frac{1}{3}} + 2^{-\frac{1}{3}} i}} \simeq 0.794 + 0.794 i$$

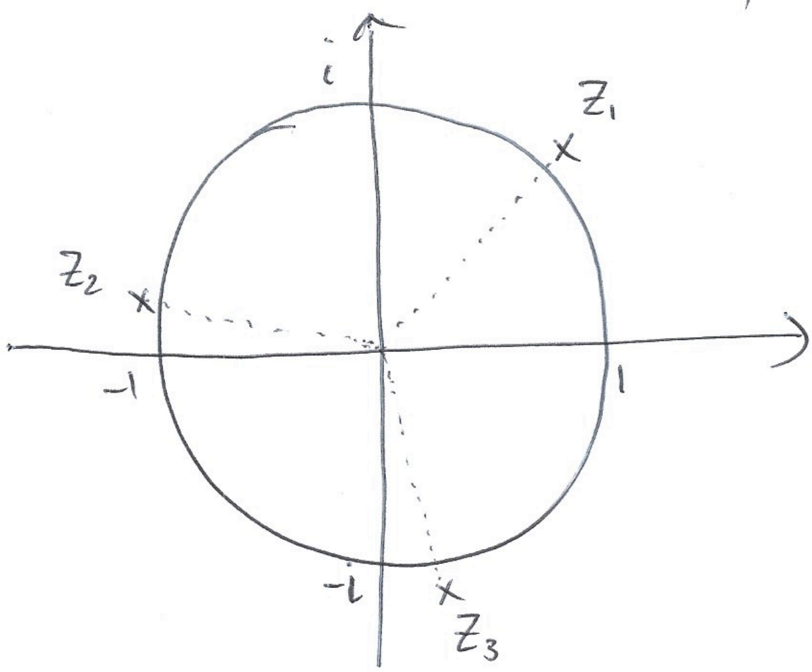
$$\boxed{\begin{aligned} &2^{\frac{1}{6}} \cdot \frac{\sqrt{2}}{2} = \\ &2^{\frac{1}{6}+\frac{1}{2}-1} = 2^{-\frac{1}{3}} \\ &\simeq 0.794 \end{aligned}}$$

$$z_2 = 2^{\frac{1}{6}} \cdot e^{\frac{3\pi}{4\cdot 3}i + \frac{2\pi}{3}i} = 2^{\frac{1}{6}} e^{\frac{11}{12}\pi i}$$

$$= 2^{\frac{1}{6}} \cdot \left(\cos \frac{11}{12}\pi + i \sin \frac{11}{12}\pi \right) \simeq \underline{\underline{-1.084 + 0.291 i}}$$

$$z_3 = 2^{\frac{1}{6}} \cdot e^{\frac{3\pi}{4\cdot 3}i + \frac{4\pi}{3}i} = 2^{\frac{1}{6}} \cdot e^{\frac{19}{12}\pi i}$$

$$= 2^{\frac{1}{6}} \left(\cos \frac{19}{12}\pi + i \sin \frac{19}{12}\pi \right) \simeq \underline{\underline{0.291 + (-1.084 i)}}$$



b) $\int \frac{(2+3\ln x)^{2/3}}{x}\,dx$

$$\boxed{\begin{aligned} u &= 2+3\ln x \\ du &= 3\cdot\frac{1}{x}dx \\ \frac{du}{3} &= \frac{1}{x}dx \end{aligned}}$$

$= \int u^{2/3} \cdot \frac{du}{3}$

$= \frac{3}{5} u^{5/3} \cdot \frac{1}{3} + C = \underline{\underline{\frac{1}{5}(2+3\ln x)^{5/3} + C}}$

c) $\int \frac{x^2-2x}{(x+2)(x^2+4)}\,dx$

DELBRØK OPPSPALTING:

$$\frac{x^2-2x}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

$$= \frac{A(x^2+4) + (Bx+C)(x+2)}{(x+2)(x^2+4)}$$

$$x^2-2x = A(x^2+4)+(Bx+C)(x+2)$$

$\underline{x=-2}$: $$(-2)^2-2(-2) = A\cdot((-2)^2+4) + 0$$

$$8 = 8A \Rightarrow \underline{A=1}$$

$\underline{x=0}$: $$0 = A\cdot 4 + (C)\cdot 2$$

$$C = -2A = \underline{-2}$$

$\underline{x=1}$ $$1-2 = A\cdot(1+4) + (B+C)(1+2)$$

$$-1 = 5 + (B-2)\cdot 3$$

$$-1 = 5 + 3B - 6 \Rightarrow \underline{B=0}$$

$$\int \frac{x^2-2x}{(x+2)(x^2+4)}\,dx = \int\left(\frac{1}{x+2} - \frac{2}{x^2+4}\right)dx$$

$$= \ln|x+2| - 2\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \underline{\underline{\ln|x+2| - \tan^{-1}\left(\frac{x}{2}\right) + C}}$$

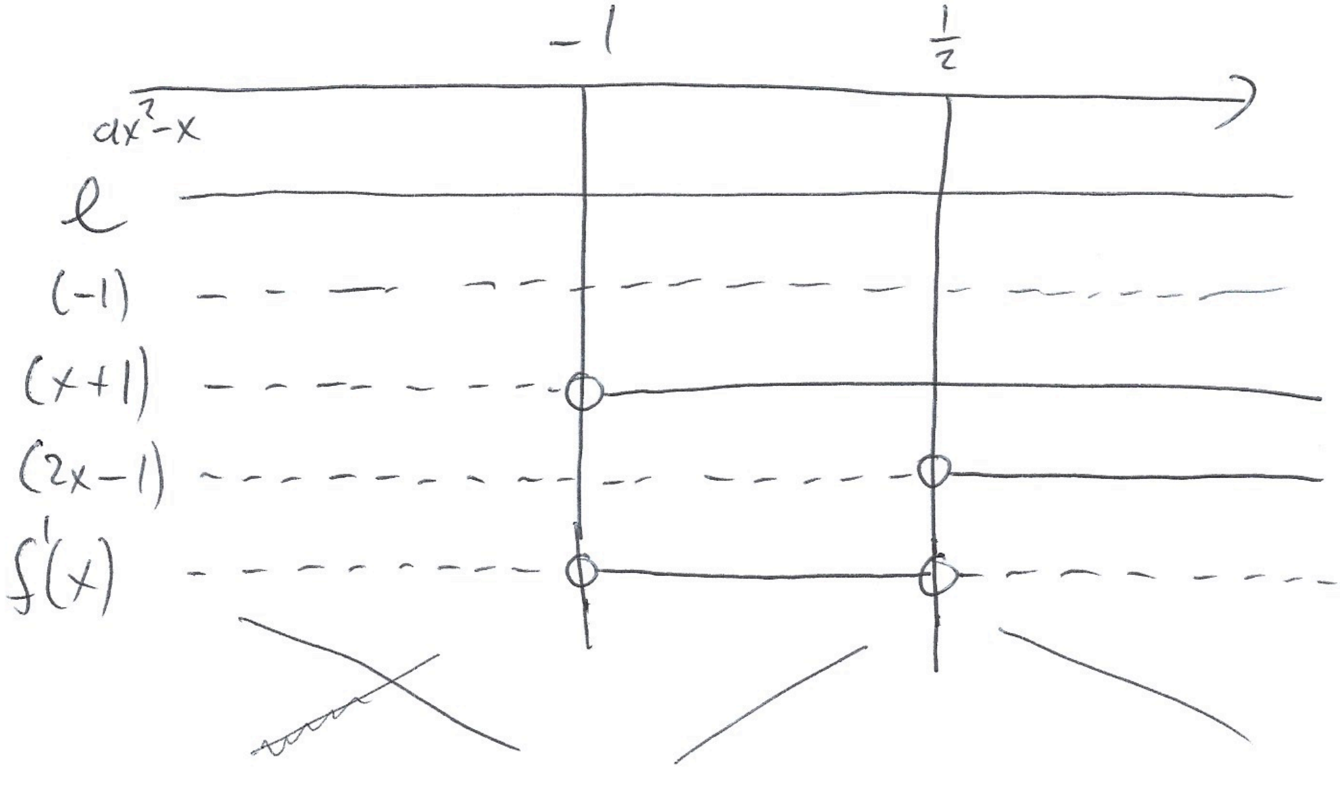
3 $f(x) = x e^{ax^2 - x}$

a) $\underline{a = -1}$

$$f'(x) = 1 \cdot e^{ax^2 - x} + x \cdot e^{ax^2 - x} \cdot (2ax - 1)$$

$$= \underline{e^{ax^2 - x}(1 + 2ax^2 - x)}$$

FOR $a = -1$: $(1 - 2x^2 - x) = \underline{-(x+1)(2x-1)}$



MONOTONT STIGENDE: $[-1, \frac{1}{2}]$

MONOTONT SYNKENDE: $\langle \leftarrow, -1]$ OG PÅ $[\frac{1}{2}, \rightarrow\rangle$

BUNNPUNKT: $x = -1$:

$y = f(-1) = (-1)e^{-(-1)^2-(-1)} = \underline{-1}$

$$\underline{\underline{(-1, -1)}}$$

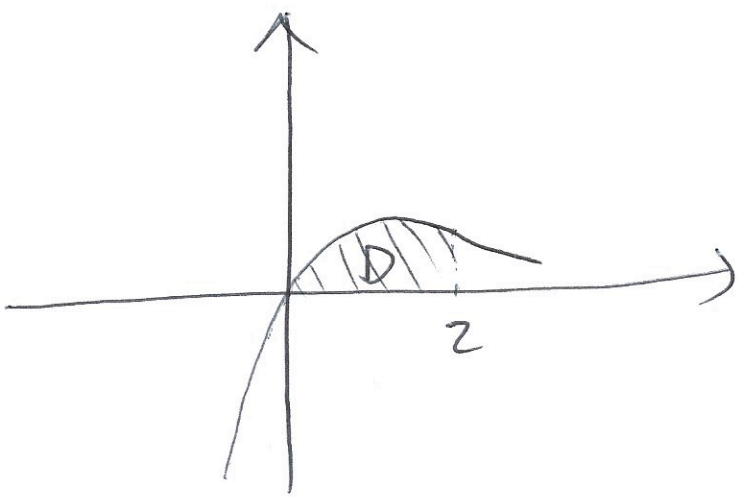
TOPPUNKT: $x = \frac{1}{2}$

$y = f\left(\frac{1}{2}\right) = \frac{1}{2} \cdot e^{-\left(\frac{1}{2}\right)^2 - \frac{1}{2}}$

$= \frac{1}{2} e^{-\frac{1}{4}-\frac{1}{2}} = \frac{1}{2} e^{-\frac{3}{4}}$

$$\underline{\underline{\left(\frac{1}{2}, \frac{1}{2}e^{-3/4}\right)}}$$

b) $\underline{a = 0}$:



$V = \pi \int_0^2 (f(x))^2 \, dx$

$$V = \pi \int_0^2 (xe^{-x})^2 dx = \int_0^2 x^2 e^{-2x} dx$$

SIDEN ALLE HJELPEMIDLER ER TILLAT KAN EN BRUKE GEOGEBRA

$$= \pi \left[-\frac{1}{8} e^{-2x} (4x^2 + 4x + 2) \right] \Bigg|_0^2 = \underline{\underline{\frac{\pi}{4}(1 - 13e^{-4})}}$$

c) VISE AT $a \geq \frac{1}{8} \Rightarrow$ MONOTONT STIGENDE FOR ALLE x.

$$f'(x) = \underbrace{e^{ax^2 - x}}_{\text{POSITIVT.}} \cdot \underline{(1 + 2ax^2 - x)}$$

MÅ SJEKKE DENNE.

NULLPUNKT?

$$1 + 2ax^2 - x = 0$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 2a \cdot 1}}{2 \cdot 2a} = \frac{1 \pm \sqrt{1 - 8a}}{4a}$$

$\underline{a = \frac{1}{8}} \Rightarrow$ ETT NULLPUNKT: $\underline{x = 2}$

$$\Rightarrow f'(x) = e^{ax^2 - x} \cdot \frac{1}{8}(x - 2)^2 \geq 0$$

FOR $a > \frac{1}{8}$: INGEN NULLPUNKT.

$$\Rightarrow (1 + 2ax^2 - x) > 0$$

$\Rightarrow f'(x) > 0$ FOR ALLE X.

DVS: $a \geq \frac{1}{8} \Rightarrow f'(x) \geq 0$

$\Rightarrow f(x)$ MONOTONT STIGENDE

(4) a) $y' + 2xy = e^{-x^2} \sin x$

$$\mu = e^{\int 2x\,dx} = \underline{e^{x^2}}$$

$$\left(y \cdot e^{x^2}\right)' = e^{-x^2} \sin x \cdot e^{x^2} = \sin x$$

$$y e^{x^2} = \int \sin x\,dx = -\cos x + C$$

$$\underline{\underline{y = (-\cos x + C)e^{-x^2}}}$$

b) $y'' + 4y' + 5y = x^2 + 2$

HOM. LIGN.

$$y'' + 4y' + 5y = 0$$

KAR. LIGN:

$$r^2 + 4r + 5 = 0$$

$$r = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 5}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \underline{-2 \pm i}$$

$$y_h = e^{-2x}(A\cos x + B\sin x)$$

PARTIKULÆR:

$$y_p = ax^2 + bx + c$$

$$y_p' = 2ax + b$$

$$y_p'' = 2a$$

$$y_p'' + 4y_p' + 5y_p = 2a + 4(2ax + b) + 5(ax^2 + bx + c)$$

$$= x^2(5a) + x(8a + 5b) + (2a + 4b + 5c)$$

$$= x^2 \qquad\qquad + 2$$

$$5a = 1 \Rightarrow a = \frac{1}{5}$$

$$8a + 5b = 0 \Rightarrow b = -\frac{8a}{5} = -\frac{8}{25}$$

$$2a + 4b + 5c = 2 \Rightarrow c = \frac{2 - 2a - 4b}{5} = \frac{72}{125}$$

$$y_p = \frac{1}{5}x^2 - \frac{8}{25}x + \frac{72}{125}$$

LØSNING:

$$y = e^{-2x}(A\cos x + B\sin x) + \frac{1}{5}x^2 - \frac{8}{25}x + \frac{72}{125}$$

c) $g(x) = x^3 + \frac{1}{12x}$

BUELENGDEN:

$$L = \int_1^3 \sqrt{1 + [g'(x)]^2}\, dx$$

$$\text{INTEGRAND} = \sqrt{1 + [g'(x)]^2} = \sqrt{1 + \left[3x^2 - \frac{1}{12x^2}\right]^2}$$

$$= \left(1 + 9x^4 - 2 \cdot 3\cancel{x^2} \cdot \frac{1}{12\cancel{x^2}} + \left(\frac{1}{12x^2}\right)^2\right)^{\frac{1}{2}}$$

$$= \left((3x^2)^2 + \frac{1}{2} + \left(\frac{1}{12x^2}\right)^2\right)^{\frac{1}{2}} = 3x^2 + \frac{1}{12x^2}$$

$$L = \int_1^3 \left(3x^2 + \frac{1}{12x^2}\right) dx = \left[x^3 - \frac{1}{12x}\right]_1^3$$

$$= 3^3 - \frac{1}{12 \cdot 3} - \left(1^3 - \frac{1}{12}\right) = \underline{\underline{\frac{469}{18}}}$$

d) $\displaystyle\int \frac{\cos\theta\, d\theta}{5+3\cos\theta}$

$x = \tan\frac{\theta}{2}$

$\theta = 2\tan^{-1}x$

$d\theta = \frac{2}{1+x^2}dx$

$\cos\theta = \frac{1-x^2}{1+x^2}$

$$= \int \frac{\frac{1-x^2}{1+x^2}\cdot\frac{2}{1+x^2}dx}{5+3\cdot\frac{1-x^2}{x^2+1}}$$

$$= \int \frac{2(1-x^2)\quad dx}{[5(1+x^2)+3(1-x^2)](1+x^2)}$$

$$= \int \frac{2(1-x^2)}{(8+2x^2)(1+x^2)}dx = \int \frac{1-x^2}{(4+x^2)(1+x^2)}dx$$

(DELBRØKOPPSPALTING).

$$= \int \left(\frac{\frac{2}{3}}{x^2+1} - \frac{\frac{5}{3}}{4+x^2}\right)dx$$

$$= \frac{2}{3}\tan^{-1}x - \frac{5}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= \frac{2}{3}\cdot\frac{\theta}{2} - \frac{5}{3}\cdot\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\tan\frac{\theta}{2}\right) + C$$

$$= \frac{1}{3}\left(\theta - \frac{5}{2}\tan^{-1}\left(\frac{1}{2}\tan\frac{\theta}{2}\right)\right) + C$$

⑤ a) I) $\lim_{x\to\infty} \frac{\ln(x^2+1)}{x+1}$ "$\frac{\infty}{\infty}$"

$\overset{l'H}{=} \lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{1} = \underline{\underline{0}}$

II) $\lim_{x\to\infty} \frac{x}{\sqrt{9x^2+a}+4} = \lim_{x\to\infty} \frac{x}{\sqrt{9x^2}}$

$= \underline{\underline{\frac{1}{3}}}$

b) $f(x) = 2+3\tan^{-1}x$

$f'(x) = \frac{3}{1+x^2}$

$f'(0) = \frac{3}{1+0} = \underline{3}$, $f(0) = 2+3\tan^{-1}0 = \underline{2}$

TANGENT: $y-2 = 3(x-0)$

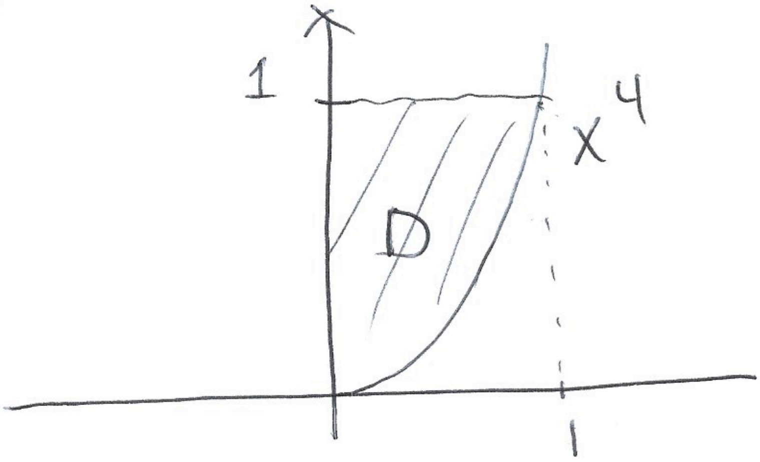
$\Rightarrow \underline{\underline{y = 3x+2}}$

NORMALEN: $y-2=-\frac{1}{3}(x-0)$

$$y=-\frac{1}{3}x+2$$

6

a)



$$V = 2\pi \int_0^1 x\left(1 - f(x)\right)dx = 2\pi \int_0^1 x(1 - x^4)\,dx$$

$$= 2\pi \left[\frac{1}{2}x^2 - \frac{1}{6}x^6\right]_0^1 = 2\pi\left(\frac{1}{2} - \frac{1}{6} - (0 - 0)\right)$$

$$= \frac{4\pi}{6} = \underline{\underline{\frac{2\pi}{3}}}$$

b) STRØMNINGSHASTIGHET $= + \dfrac{dV}{dt}$.

VED HØYDEN $h = x^4 \Rightarrow x = \sqrt[4]{h}$.

$$V(h) = 2\pi\left[\frac{1}{2}hx^2 - \frac{1}{6}\cancel{x}^6\right]_0^{h^{\frac{1}{4}}} = \frac{2\pi}{3}h^{\frac{6}{4}} = \frac{2\pi}{3}h^{\frac{3}{2}}$$

$$\frac{dV}{dt} = \frac{2\pi}{3} \cdot \frac{3}{2} \cdot h^{\frac{1}{2}} \cdot \frac{dh}{dt} = \pi h^{1/2} \frac{dh}{dt}$$

$$\pi h^{1/2} \frac{dh}{dt} = +0.007$$

VED $h = 0.5$:

$$\left.\frac{dh}{dt}\right|_{h=0.5} = + \frac{0.007}{\pi \cdot \sqrt{0.5}} = \underline{\underline{+0.00315}}$$

LØSER:

$$h^{1/2} \frac{dh}{dt} = + \frac{0.007}{\pi}$$

$$\int h^{1/2} \, dh = + \int \frac{0.007}{\pi} \, dt$$

$$\frac{2}{3} h^{3/2} = + \frac{0.007}{\pi} t + C$$

$t=0 \Rightarrow h = 0 \Rightarrow \cancel{C \neq \frac{2}{3}}$ $C=0.$

$\Rightarrow h=0$ NÅR $t = \frac{2}{3} \cdot \frac{\pi}{0.007} = 299$ (min)

DVS. ETTER 4 timer 59 min ER

STAMPEN FULL