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Reservoir Engineering PET120

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Petrophysics

Intro (summary)

- Golden zone (60-120 C)
- Migration is assumed to have started at burial depths where T is raised above 150 C
- Hydraulic fracturing of the caprock due to increasingly higher pore pressure, makes oil and gas escape to lower depths.
- Migration to low depth where T<60 C, will normally not occur due to less effective cementation process and/or microorganisms' affinity for hydrocarbons
- Experiments have shown that gas is generally generated at higher depths than oil
- Average duration of the production phase is 10-30 years
  drilling additional wells to stimulate production can increase lifetime
- P&A average duration is 5-10 years

Porosity:

$$Porosity: \phi = \frac{Total\ volume\ of\ pores}{Total\ volume\ of\ rock\ (Bulk\ Volume)}$$

$$\phi = \frac{V_p}{V_b} = \frac{V_b - V_{matrix}}{V_b}$$

$$Ideal\ gas\ law: PV = nRT \rightarrow Boyles\ law: P_1V_1 = P_2V_2\ at\ T = consant$$

$$Porevolume: V_p = \frac{m_{saturated} - m_{dry}}{\rho_l}, \quad V = \frac{mass\ (m)}{density\ (\rho)}$$

- Depends on packing arrangement of rock: Consolidate (particles are cemented/compacted), Unconsolidated (particles are loosely arranged like sand)

Saturation:

$$V_p = V_0 + V_g + V_w$$

$$S_i = \frac{V_i}{V_p}, \qquad i = water, oil\ etc$$

$$Oil\ Recovery\ Factor: RF = \frac{Amount\ of\ recoverd\ oil}{Original\ Oil\ in\ Place}$$

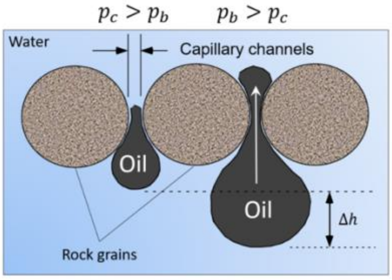
$$Residual\ oil\ saturation: S_{or} = (1 - S_{wi}) - \frac{V_{op}}{V_p}$$

- S_{wi} – irreducible water from 5-40%
- Used for Original Oil in Place (OOIP) and Original Gas in Place (OGIP)
- GOC- Gas-Oil Contact
- OWC- Oil-Water Contact
- FWL – Free water level

Capillary Pressure/Reservoir Pressure

$Capillary\ Pressure: p_c = \dfrac{2\sigma\cos\theta}{r_c}$, $\quad p_b = (\rho_w - \rho_o)gh$

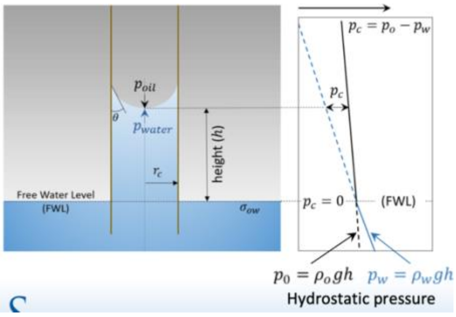
$\sigma -$ interfacial tension(IFT)
$\theta -$ contact angle
(measure of rock wettability
$r_c -$ capillary radius
$p_b -$ buoyancy pressure
$\left(\dfrac{dp}{dD}\right)_w -$ pressure gradient
$-$ rate of change with depth
$D -$ depth
$C -$ constant accountign for devation
normal hyrdo static pressure



Equilibrium in a capillary tube for oil-water system:

$$p_c = \frac{2\sigma_{ow}\cos\theta}{r_c} \quad or$$
$$p_c = p_o - p_w = (\rho_w - \rho_o)gh$$
$at\ certain\ h, p_o\ and\ p_w$:
$$p_0 = -\rho_o gh, p_w = -\rho_w gh$$

- Negative sign indicates that Hydro Static Pressure resist Displacement



- $p_{c(FWL)} = p_o - p_w = 0 at\ FWL\ p_o = p_w,\ p_{c(GOC)} = p_{gas} - p_o = 0 \rightarrow p_{gas} = p_{oil}$

Pressure distribution in reservoirs:

The total pressure at any reservoir depth: <u>overburden pressure</u> is the sum of rock matrix pressure pore-fluid pressure.

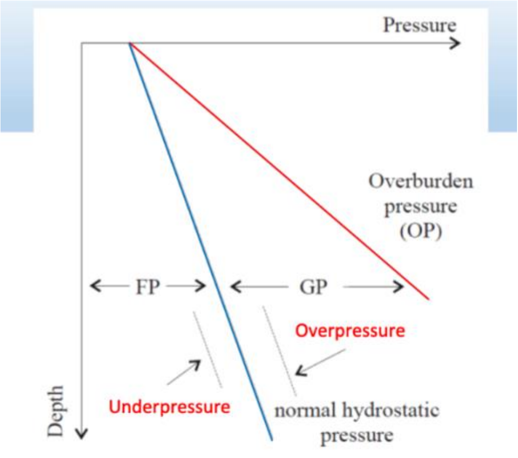
$$p_{ov} = p_m + p_f$$

Fluid pressure (pore pressure) is determined relative to normal pressure (HSP)

$$p_w(D) = \left(\frac{dp}{dD}\right)_w D + 14.7 + C\ [pisa]$$

Abnormal reservoir pressure:

- Normal HSP curve is taken as a reference
- Pore pressure in excess of normal pressure is overpressure, under is under pressure
- Overburden pressure is constant at any particular depth→the differential $dp_{ov} = 0$
- $dp_f = -dp_m$
- Any reduction in fluid pressure dp_f, as it occurs during production, will lead to a corresponding increase in the grain (matrix) pressure dp_m



Reservoir pressure terminology
(FP – fluid pressure, GP – grain pressure)

Capillary Pressure calculations:

- The difference in phase pressure at the same formation depth refers to capillary pressure or buoyancy.
  e.g. oil-water system at certain depth (h)
  $$p_{c(h)} = p_{o(h)} - p_{w(h)} = (\rho_w - \rho_o)gh = p_b$$
- Capillary pressure is additive value:
  $$(p_c)_{gw} = (p_c)_{ow} + (p_c)_{go}$$
- Threshold capillary pressure is minimum pressure required for oil to displace formation water, which initially saturates the porous space

Wettability:

$$\cos\theta = \frac{\sigma_{os} - \sigma_{ws}}{\sigma_{ow}}$$

$$p_c = p_{nwt} - p_{wt}, p_c = p_{oil} - p_{water}$$

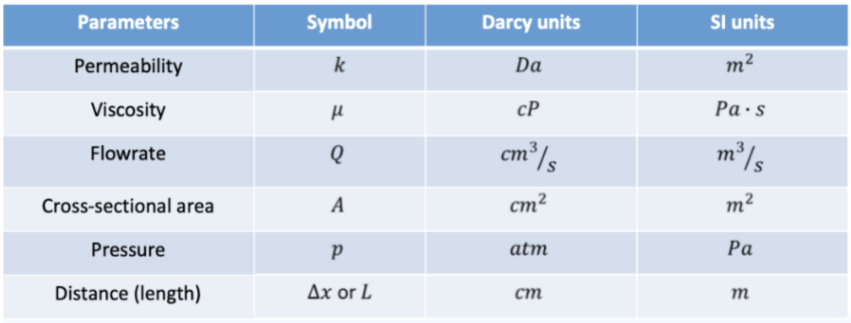
$$p_c = \sigma \cdot (\frac{1}{R_1} + \frac{1}{R_2}), p_o - p_w = \sigma_{ow} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right), \mathrm{R}_1 = R_2 = \frac{r_c}{\cos\theta}$$

$$\frac{p_{c,lab}}{(\sigma\cos\theta)_{lab}} = \frac{p_{c,res}}{(\sigma\cos\theta)_{res}}, p_{c,res} = \frac{p_{c,lab}\sigma_{res}}{\sigma_{lab}}, \ h_{res} = \frac{p_{c,res}}{g(\rho_{wet} - \rho_{nonwet})}$$

Permeability:

Darcy's law for linear and horizontal flow:

$$k = \frac{Q\mu}{A}\frac{L}{p_1 - p_2}$$



Parameters	Symbol	Darcy units	SI units
Permeability	k	Da	m^2
Viscosity	μ	cP	$Pa \cdot s$
Flowrate	Q	$cm^3/_s$	$m^3/_s$
Cross-sectional area	A	cm^2	m^2
Pressure	p	atm	Pa
Distance (length)	Δx or L	cm	m

- **Absolute permeability:** when the porous medium is completely saturated with one fluid
- **Effective permeability:** the ability of fluid to flow in the presence of other fluid in porous medium (e.g. effective permeability of oil in an oil-water system)
- **Relative permeability:** the ratio of effective permeability of a particular fluid at a particular saturation to absolute permeability of that fluid at total saturation.

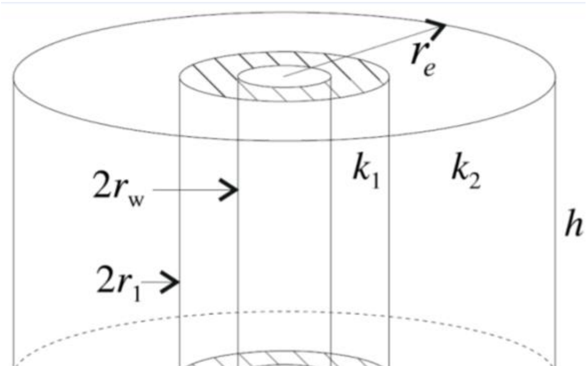
Linear flow through layered bed:

$$\bar{k} = \frac{\sum_{j=1}^{n} k_j h_j}{\sum_{j=1}^{n} h_j}$$

Linear flow through parallel layers:

$$\bar{k} = \frac{\sum_{j=1}^{n} l_j}{\sum_{j=1}^{n} \frac{l_j}{k_j}}$$

Linear flow through stack of cylindrical layers:



$$\bar{k} = \frac{\ln\left(\frac{r_e}{r_w}\right)}{\sum_j^n (\ln \frac{r_j}{r_{j-1}}) \frac{1}{k_j}}$$

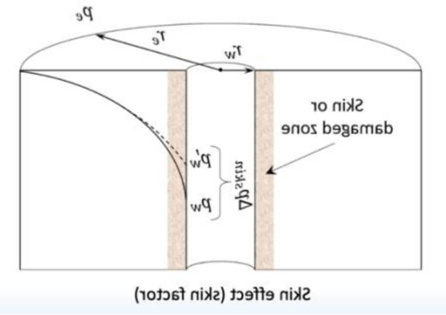
Radial flow & Fluid viscosity

Radial flow:
For a radial system with linear flow in a cylindrical reservoir where flow across the formation is horizontal and fluid moves radially towards the well bore:

$$q = \frac{2\pi hk}{\mu} \cdot \frac{p_e - P_w}{\ln \frac{r_e}{r_w}}, h - pay\ zone\ thickness$$

$$p_e = p_w + \frac{q\mu}{2\pi kh} \ln \frac{r_e}{r_w}$$

Skin effect:

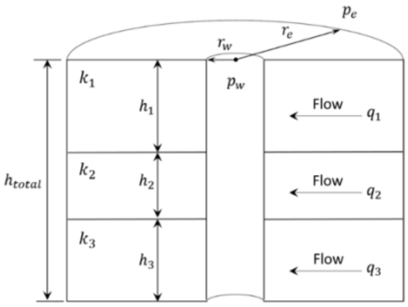


$$\Delta p_{skin} = ideal\ pressure\ (p'_w - acutal\ pressure\ (p_w)$$

$$\Delta p_{skin} = \frac{q\mu B}{2\pi kh} \cdot S, s - skin\ factor,$$

$$B - formation\ volume\ factor$$

Radial flow through stack of layered parallel layers:



$$\bar{k} = \frac{\sum_{j=1}^n k_j h_j}{\sum_{j=1}^n h_j}, \quad q = \frac{\sum_{j=1}^n k_j h_j}{\sum_{j=1}^n h_j} \cdot \frac{2\pi h}{\mu} \cdot \frac{p_e - p_w}{\ln \frac{r_e}{r_w}}$$

Fluid viscosity

Definition: Fore required to move upper moving plane can be expressed by:

$F = \frac{\mu A v_x}{y} \rightarrow \frac{F}{A}$ $or\ \tau = \mu \frac{dv_x}{dy}, \mu = \frac{\tau}{\frac{dv_x}{dy}}$ Shear stress: $\tau = \frac{F}{A}\left[N \cdot \frac{s}{m^2}\right]$ Shear rate: $\frac{dv_x}{dy} = \frac{m}{s \cdot m} = \frac{1}{m}$

$$\mu \rightarrow \left[\frac{N \cdot s}{m^2}\right] \rightarrow [Pa \cdot s]\ 1Pa.s = 10\ P\ (poise) = 10^3\ cP\ (centipoise)$$

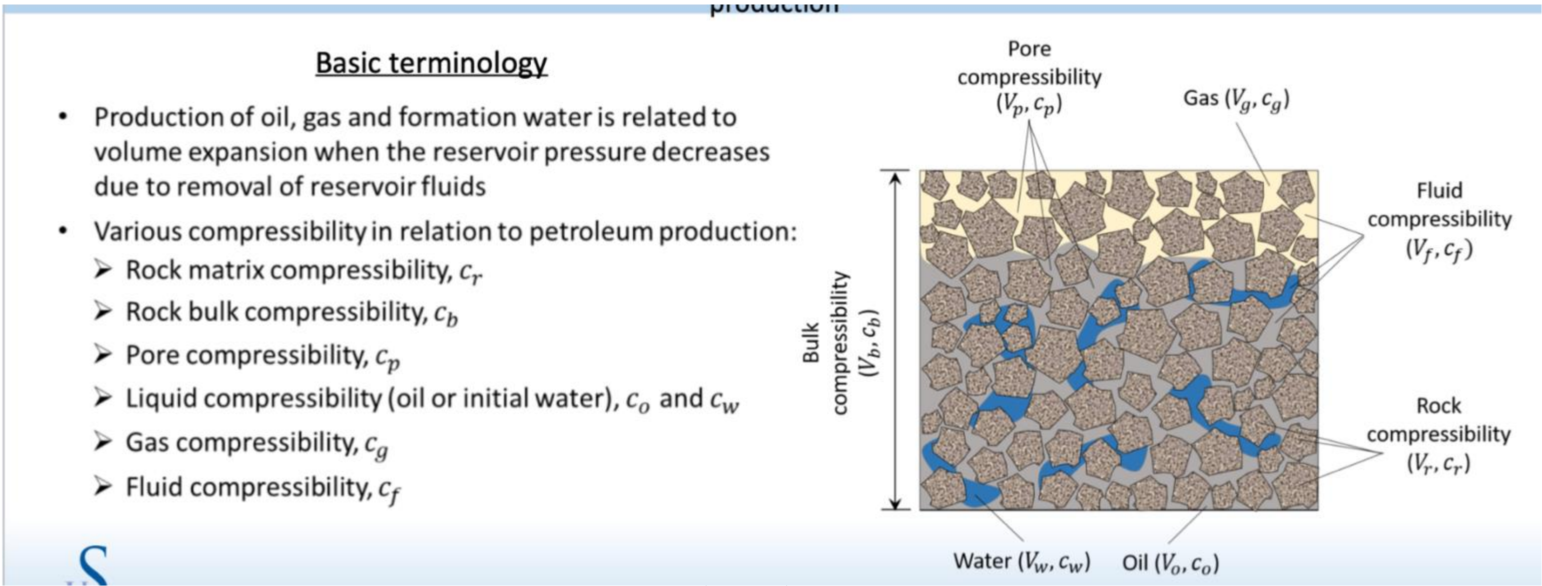
Viscous flow rate: Poiseuille's equation:

$$v_x = \frac{r^2}{8\mu}\frac{\Delta p}{l} \rightarrow \quad q = A\frac{r^2}{8\mu}\frac{\Delta p}{l}\ , \quad k = \frac{\phi r^2}{8}$$

Compressibility:

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T,$$

$$\left(\frac{\partial V}{\partial p}\right)_T - change\ in\ volume\ of\ material\ related\ to\ a\ unit\ change\ in\ pressure\ (isthermal\ process)$$



Basic terminology

- Production of oil, gas and formation water is related to volume expansion when the reservoir pressure decreases due to removal of reservoir fluids
- Various compressibility in relation to petroleum production:
  - Rock matrix compressibility, c_r
  - Rock bulk compressibility, c_b
  - Pore compressibility, c_p
  - Liquid compressibility (oil or initial water), c_o and c_w
  - Gas compressibility, c_g
  - Fluid compressibility, c_f

$$c_{total} = c_p + c_f = \frac{c_b - (1+\phi)c_r}{\phi} + \left(c_w S_w + c_o S_o + c_g S_g\right)$$

Compressibility of ideal gas:

$$pV = nRT,\ \ c_g = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = \frac{1}{p}$$

For real gases:

$$pV = ZnRT,\ \ c_g = \frac{1}{p} - \frac{1}{z}\frac{dz}{dp}$$

Gas permeability:

$$k = \frac{q_0 \mu}{A}\frac{p_0}{\bar{p}}\frac{\Delta l}{\Delta p}$$

Capillary number & Relative permeability

$$\Delta p_{viscous} = \Delta p_{oil} + \Delta p_{water} = \frac{q}{Ak}\left(\mu_w x + \mu_o (L-x)\right), \Delta p_{capillary} = \frac{2\{sigma_{ow}\cos\theta)}{r_c}$$

$$\frac{\Delta p_{viscous}}{\Delta p_{capillary}} = 4\left[\frac{v\mu_o}{\sigma_{ow}\cos\theta}\frac{x}{r_c} + \frac{v\mu_w}{\sigma_{ow}\cos\theta}\frac{L-x}{r_c}\right]$$, v- fluid velocity in pores, k=$\frac{r^2}{8}$

The only dynamical term in the equation above: $\frac{v\mu}{\sigma\cos\theta} = N_c$- capillary number.
An increase in the capillary number implies increase in oil recovery.

Relative permeability:

$$k_{rj} = \frac{k_{ej}}{base\ permeability}$$

Pressure, Volume & Temperature

Intro:

$P_{sc} = 1bar\ (14.7\ psi)\ \ T_{sc} = 15^0C\ (60^0F)$,

Sub index: res (reservoir conditions), SC (standard condition, atmospheric conditions)

Flooding rate: $Q = \frac{\Delta V}{\Delta t}$, Mole fractions: $z_i = \frac{n_i}{\sum n_j} = \frac{number\ of\ moles\ of\ component\ i}{total\ numer\ of\ moles}$

Weight fractions: $w_i = \frac{w_i}{\sum w_i} = \frac{weight\ of\ component\ i}{total\ weight}$

Gibbs phase rule: F=C+2-P, F- number of degrees of freedom, C- number of components, P- number of phases in equilibrium.

Ex. H20: C=1, P=1 (liquid) P=2 (slush) P=3 (triple point)

Critical point:

The critical temperature/pressure are the maximum P and T at which a pure compound can form coexisting phases.

Classification of Reservoir fluid, reservoir temperature, GOR and oil gravity

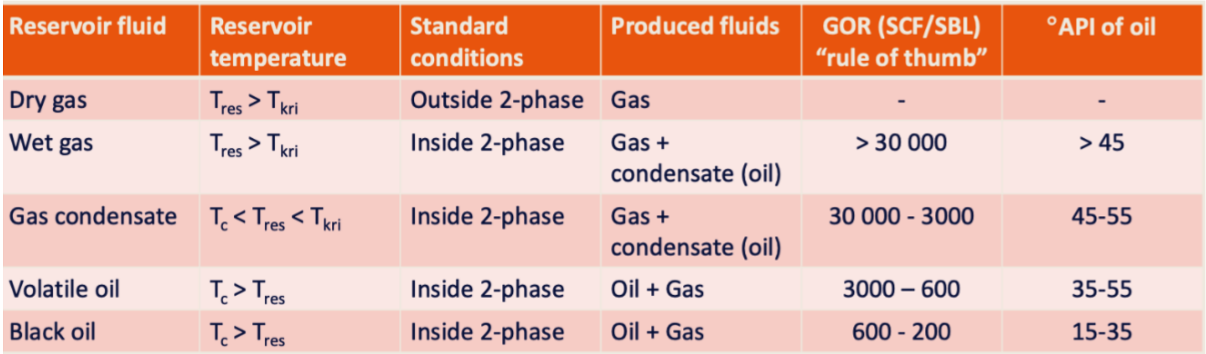
- A reservoir contains gas if $T_{res} > T_c$
- A reservoir contains oil if $T_{res} < T_c$
- Retrograde condensation of heavier oil components in reservoir when
$$T_c < T_{res} < T_{kri}$$
- No liquid forms if $P_{res} > P_{kri}$

GOR:

- The number of cubic feet of the associated gas produced at SC per barrel of Stock tank oil in field units. $\frac{SCF}{SBL}\ \ \frac{Sm^3_{gas}}{Sm^3_{oil}}$

Stock tank oil gravity:

$$API = \frac{141.5}{S_o} - 131.5 = \frac{141.5}{\gamma_0} - 131.5$$



Reservoir fluid	Reservoir temperature	Standard conditions	Produced fluids	GOR (SCF/SBL) "rule of thumb"	°API of oil
Dry gas	$T_{res} > T_{kri}$	Outside 2-phase	Gas	-	-
Wet gas	$T_{res} > T_{kri}$	Inside 2-phase	Gas + condensate (oil)	> 30 000	> 45
Gas condensate	$T_c < T_{res} < T_{kri}$	Inside 2-phase	Gas + condensate (oil)	30 000 - 3000	45-55
Volatile oil	$T_c > T_{res}$	Inside 2-phase	Oil + Gas	3000 – 600	35-55
Black oil	$T_c > T_{res}$	Inside 2-phase	Oil + Gas	600 - 200	15-35

Gas specific gravity (γ_g):

$$\gamma_g = \frac{MW_m}{MW_{air}}$$

Gas density:

$$\rho_g = \frac{m}{v} = \frac{PMW_m}{ZRT}, \qquad m = n * Mw, \qquad MW_m - mean\ molecular\ weight$$

Gas formation volume factor:

$B_g = \frac{V_{res}}{V_{sc}} = \frac{\frac{z_{res}nRT_{res}}{P_{res}}}{\frac{z_{sc}nRT_{sc}}{P_{sc}}}$, for the same mass, nR cancel out, and $z_{sc} \sim 1, T_{sc}$ & P_{sc} are constants:

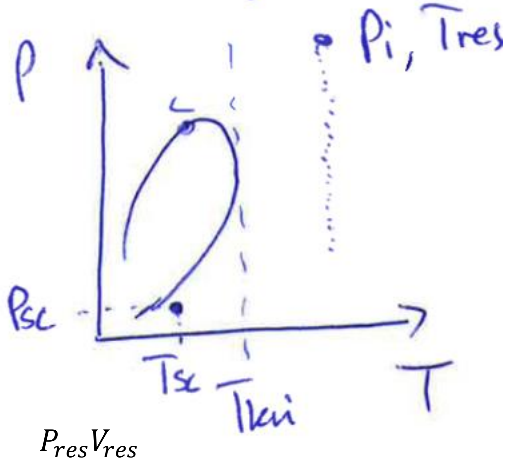
$$B_g = 0.0283 \frac{z_{res}T_{res}}{P_{res}}\ ft^3/SCF$$

Material balance:

Dry gas:

What is the volume of produced gas, G_p, at SC, given initial reservoir pressure P_{ic} and current reservoir pressure P_{res}

Dry gas PT-diagram



Assumptions: enclosed reservoir, constant reservoir volume, HCPV=constant, $HCPV = V_b \cdot \phi(1 - S_{wi})$

- T= constant under depressurization

$$PV = ZnRT \rightarrow n = \frac{PV}{ZRT}$$

Mass balance: $P_{ic} \rightarrow P_{res}$

$n_{prod,SC} = n_{ic} - n_{res}$, $G_p = n_{prod}\, V_m$

$$\left(\frac{PV}{ZRT}\right)_{sc} = \left(\frac{PV}{ZRT}\right)_{ic} - \left(\frac{PV}{ZRT}\right)_{res}, \frac{P_{sc}V_{sc}}{Z_{sc}RT_{sc}} = \frac{P_{ic}V_{ic}}{Z_{ic}RT_{ic}} - \frac{P_{res}V_{res}}{Z_{res}RT_{res}}$$

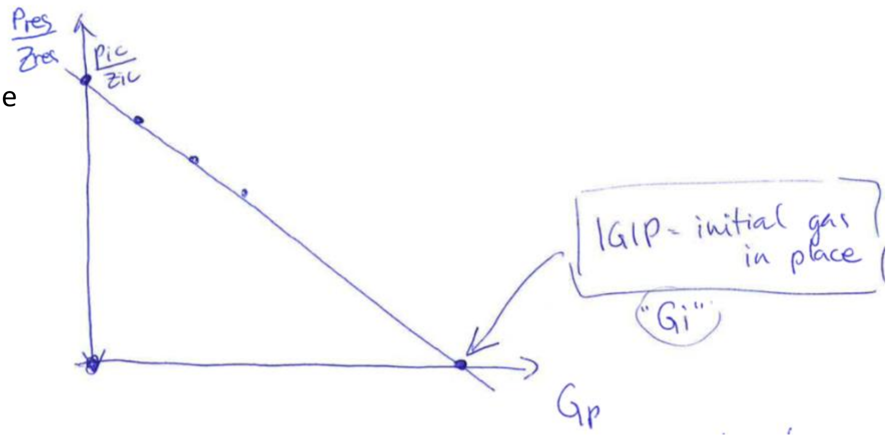
- $V_{sc} = G_p$
- $V_{ic} = V_{res}$ enclosed reservoir
- $T_{ic} = T_{res}$ constant T
- $Z_{sc} = 1$ all gases act ideally at SC

Solve with respect to $\frac{P_{res}}{z_{res}} = \frac{P_{ic}}{z_{ic}} - \frac{P_{sc}T_{ic}}{V_{ic}T_{sc}} G_p$ and plot against GP

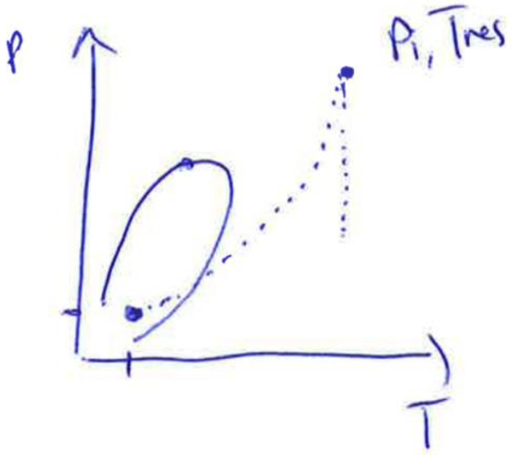
$IGIP = n_i \cdot V_m$

Initial Gas in Place=

Initial amount of moles * Molar volume



Wet gas PT diagram:



Wet gas:
- constant T, depressurization.

Under production to SC both P and T decreases and the reservoir fluid and moves into Two-phase-envelope.
Material balance calculations identical to dry gas, but only after produced STO and water is converted to gas equivalents.

How to calculate oil' gas equivalent volume
- Starting point in 1 SBL STO

How many gas equivalents (GE) in SCF
Use Real gas law

$$GE_{STO} = V_{sc} = \frac{z_{sc} n R T_{sc}}{P_{sc}} = n \cdot \frac{Z_{sc} R T_{sc}}{P_{sc}} = n \cdot \frac{10.732(60+460)^0 R}{14.7}$$

Number of moles in 1bbl STO
If γ_{STO} or M_{STO} are given, then $n_{STO} = \frac{m_{STO}}{M_{STO}}$,

$$\gamma_{STO} = \frac{\rho_o}{\rho_w} \rightarrow \rho_o = \gamma_{STO} \cdot \rho_w, \;\; m_o = \rho_0 V_o$$

$$n_{STO} = \frac{m_{STO}}{M_{STO}} = \frac{\rho_{STO} V_{STO}}{M_{STO}} = \frac{\gamma_{STO} \rho_w V_{STO}}{M_{STO}} = \frac{\gamma_{STO} \cdot \frac{1g}{cm^3} \cdot 62.43 \left(\frac{\frac{lb}{ft^3}}{\frac{g}{cm^3}} \right) \cdot 5.615 \frac{ft^3}{bbl}}{M_{STO} \left(\frac{lb}{lbmol} \right)}$$

$$= 350.54 \frac{\gamma_{STO}}{M_{STO}} \;\; lb\, mol$$

$$GE_{STO} = V_{SC} = \frac{\gamma_{STO}}{M_{STO}} \cdot 350.52 \cdot \frac{10.732 \cdot 520}{14.7} \approx 133000 \frac{\gamma_{STO}}{M_{STO}} \left[\frac{SCF}{SBL} \right] \; or$$

$$\frac{133000}{5.615} \frac{\gamma_{STO}}{M_{STO}} \left[\frac{Sm^3}{Sm^3} \right]$$

Produced oil Gas equivalent volume

$$total\; GE_{STO}[SCF] = GE_{STO} \left[\frac{SCF}{SBL} \right] \cdot V_{STO}[SBL]$$

If γ_{STO} is given, not M_{STO}, we can use Cragoe's formula to determine M_{STO}

$$M_{STO} = \frac{6084}{°API - 5.9} \rightarrow °API = \frac{141}{\gamma_o} - 131.5, \gamma_o = \frac{\rho_o}{\rho_w}$$

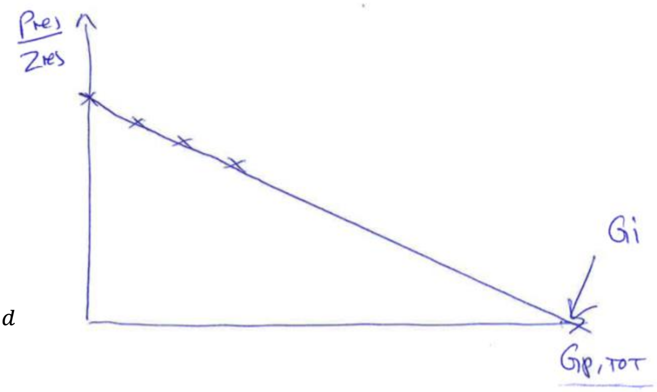
If water is produced from the reservoir, we can find water gas equivalent volume:

$$\gamma_w = 1 \quad M_w = 18 \rightarrow GE_w = 133000 \frac{\gamma_w}{M_w} \approx 133000 \cdot \frac{1}{18} = 7389\; SCF/SBL$$

Material balance for a wet gas:
Volume of reservoir fluid produced / Total well fluid as a gas produced

$$G_{p,tot\,(SC)} = V_{g,SC} + V_{STO} \cdot GE_{STO} + V_w \cdot GE_W, \; B_g = \frac{V_{res}}{V_{sc}} \rightarrow V_{res} = G_{p,tot} \cdot B_g$$

To determine initial well fluid as gas, $G_i(SC)$: Plot $\frac{P_{res}}{z_{res}}$ against $G_{p,tot}$



$G_i = IGIP + IOIP \cdot GE_{STO}$

To calculate Production of oil and gas:

1. $n_{prod} = n_{ic} - n_{res}$, $PV = ZnRT$
2. $V_{g,SC} = n_{prod} \cdot V \cdot V_m$, $V_{STO} = \frac{n_{prod} L M_{STO}}{\rho_{STO}}$

   mole fraction of L(liquid) and V(gas) from n_{prod}

   $GOR = \frac{V_{g,SC}}{V_{STO}}$, $\rho = \frac{m}{V}$, $V = \frac{m}{\rho} = \frac{nM}{\rho}$

   IGIP and IOIP volume at SC:

   Amount of initial reservoir fluid: n_i

   $IGIP = n_i \cdot V \cdot V_m$

   $$IOIP = \frac{n_i \cdot L \cdot M_{STO}}{\rho_{STO}}$$

   $$GOR_{initial} = \frac{IGIP}{IOIP}$$

Gas condensate: lecture 5 050220 and lecture 6

Reservoir Condition phase behavior (Equilibrium calculations)

$K_i = \frac{y_i}{x_i}$, K_i- physical equilibrium constant giving ratio of component in gas and liquid phase

y_i- concentration of component in the vapor phase

x_i- concentration of component in the liquid phase

Defined by:

- Dalton's law for ideal gases:

$$P = P_1 + P_2 + .. + P_N = \sum P_i$$

$$P_i = \frac{n_i RT}{V}, \qquad i = 1,2..,N$$

$$\frac{PV}{RT} = n, \quad \sum n_i = \frac{(\sum P_i)V}{RT}$$

$$y_i = \frac{n_i}{n} = \frac{P_i}{P}$$

$$P_i = y_i P$$

- Raoult's law for ideal liquids:

$$x_i = \frac{P_i}{P_i * (T)}, \quad P_i = x_i P_i * (T)$$

- System equilibrium at P and T

  $P_i(vapor) = P_i(liquid), y_i P = x_i P_i * (T)$, $K_i = \frac{y_i}{x_i} = \frac{P_i(T)}{P}$

Flash calculations in lecture 7

Separator calculations:

We have j number of separators, mole fraction liquid enters separator “j”

$$(n_o)_j = 1mole \cdot L_1 \cdot L_2 \cdot L_3 \cdot \ldots \cdot L_j = \prod_{i=1}^{j} L_j$$

If Separator “k” is stock tank; mole fraction fluid that reaches the tank:

$$n_{STO} = \prod_{i=1}^{k} L_i$$

For gas: moles of gas from separator 1 to k, mole fraction:

$$\left(n_g\right)_k = 1mole \cdot V_1 + L_1V_2 + L_1L_2V_3 + .. + L_1 \cdot L_2 \cdot L_{k-1} \cdot V_k, \qquad L_0 = 1$$

$$n_g = \sum_{i=1}^{k} V_i \cdot \prod_{j=0}^{i-1} L_j$$

Given that sum of mole should equal 1 mole initial reservoir fluid:

$$n_g = 1 - n_{STO} = 1 - \prod_{i=1}^{k} L_i$$

Volume of gas Separator at SC and STO

$$V_g = number\ of\ moles\ in\ gas \cdot molar\ volume\ at\ SC\ (1\ mole\ gas)$$

$$V_g = n_g \cdot V_m = (1 - n_{STO})V_m = \left(1 - \prod_{i=1}^{k} L_i\right)V_m$$

$$V_{STO} = \frac{m_{STO}}{\rho_{STO}} = \frac{n_{STO}M_{STO}}{\rho_{STO}} = \frac{M_{STO}\prod_{i=1}^{k} L_i}{\rho_{STO}}$$

Total GOR:

$$GOR_{total} = \frac{V_g}{V_{STO}} = \frac{n_gV_m\rho_{STO}}{n_{STO}M_{STO}} = \frac{\left(1 - \prod_{i=1}^{k} L_i\right)V_m\rho_{STO}}{\prod_{i=1}^{k} L_i \cdot M_{STO}}$$

GOR for separator i:

$$GOR_i = \frac{\left(V_g\right)_i}{V_{STO}} = \frac{\left(n_g\right)_i \cdot V_m \cdot \rho_{STO}}{n_{STO} \cdot M_{STO}} = \frac{V_i\prod_{i=1}^{i-1} L_i \cdot V_m \cdot \rho_{STO}}{\prod_{i=1}^{k} L_i \cdot M_{STO}}$$

Formation Volume factor for oil B_o

- Defined as the relation between reservoir volume of oil and STO SC

$$B_o = \frac{V_o}{V_{STO}}$$

$$B_o = \frac{V_o}{V_{STO}} = \frac{\frac{n_oM_o}{\rho_o}}{\frac{n_{STO}M_{STO}}{\rho_{STO}}} = \frac{M_o\rho_{STO}}{\prod_{i=1}^{k} L_i \cdot M_{STO} \cdot \rho_o}$$

Flash calculation in lecture 8

Well Testing:

Multiphase flow: