Høsten 2021

FYS100 Mekanikk: Exam/Eksamen

You **must** put your candidate number on every sheet.

There are 4 questions. You need to answer all 4 questions for a full score.

The standard formula sheet for FYS100 Mekanikk is part of this question sheet.

Standard approved calculators are allowed.

Don't panic! Draw a diagram where relevant. State clearly the relevant physics.

The questions are also attached in Norwegian.

Good luck!

Du **må** legge kandidatnummeret ditt på hvert ark.

Det er 4 spørsmål. Du må svare på alle de fire spørsmålene for en full score.

Standardformelarket for FYS100 Mekanikk er en del av dette spørsmålet.

Standard godkjente kalkulatorer er tillatt.

Ingen panikk! Tegn et diagram der det er relevant. Angi tydelig hvilken fysikk som er relevant.

Spørsmålene er også vedlagt på engelsk.

Lykke til!

Problem 1: Electric aeroplane

A propeller plane of mass $m_p = 2.4$ tonnes is flying horizontally at an altitude of $A = 2300$m.

a) Draw a diagram showing the direction of relevant forces on the aeroplane that keep it flying at a constant height at a constant speed.

Solution: The diagram should show a weight force acting down, a lift force acting up, a thrust force acting forward in the direction of motion and a drag force acting backwards.

b) If the lift force to drag force ratio of the plane is $R = 11$, find the thrust and power needed from the motor to keep the plane flying horizontally at a constant speed of $v = 220$km/h.

Solution: By Newton's laws, the lift force has to equal the weight force for flying at a constant altitude, $L = mg$. The thrust force must equal the drag force for flying at a constant speed, $T = D$. The ratio is $R = L/D$. So $T = \frac{mg}{R} = \frac{2400*9.82}{11} = 2100$N (2 sig. fig.). The power fom the motor is $P = Tv = \frac{mg}{R}v = \frac{2400*9.82}{11} * 220 * \frac{1000}{3600} = 130$kW (2 sig. fig.)

c) The propeller is driven by an electric motor with an electric battery of mass density of $\rho_m = 2400$ kg/m^3 and an energy density when fully charged of $\rho_e = 1800$ MJ/m^3. Stating any assumptions you make, show that by considering different sizes of battery, there is a maximum distance the plane can be powered at a constant altitude and speed, given by

$$d = \frac{\rho_e}{\rho_m}\frac{R}{g}$$

Solution: The power gives the energy used per second by the engine. This can be converted into the energy used per metre by dividing by v (the force is energy used per distance.) The total distance that can be travelled is therefore given by the total energy available from the battery divided by the thrust $d = E/T$. The total energy available is given by the energy density multiplied by the volume of the battery $E = \rho_e V = \frac{\rho_e}{\rho_m} m_b$ where m_b is mass of the battery. Thus the total distance that the plane can fly is

$$d = \frac{\rho_e}{\rho_m}\frac{m_b}{T} = \frac{\rho_e}{\rho_m}\frac{m_b}{m_p}\frac{R}{g}$$

In the limit of a very light plane (or very large batteries), the mass of the battery dominates the total mass of the plane and the result

is obtained. It is not possible to fly further than this by adding more batteries.

d) Estimate the value of this distance using the values given above. How might things be different in the real world?

Solution: Putting numbers in gives $d = 840$km (2 sig. fig.) This is if all the energy in the battery is used for thrust at cruising speed. Apart from needing extra energy for takeoff and landing, one might also need extra energy for a safety margin for turbulence and head-winds, extra energy for air conditioning or cooling the motor and overcoming friction in the propeller shaft. This distance is the limit for large batteries dominating the total mass of the plane, which is also not very realistic.

Problem 2: Icy road

a) A front-wheel-drive car of mass $m = 1200$kg is driving on an icy road. The coefficient of static friction between the tyres and the icy road is $\mu_s = 0.20$ and the coefficient of kinetic friction is $\mu_k = 0.15$. Assuming the weight of the car is evenly distributed on all four wheels, what is the maximum speed the car can drive around a horizontal curve of radius $r = 15$m without any of the wheels sliding?

Solution: To move around the bend, the car must have a centripetal acceleration of $a = \frac{v^2}{r}$. The force that provides this acceleration is ultimately given by the friction between the tyres and the road. This cannot exceed the maximum static friction between the tyres and the road or else the wheels will slide.

The maximum static frictional force is given by $f_{s,max} = \mu_s N$ where N is the normal force. By assumption, the weight of the car is equally supported by all four wheels, so by Newton's first law the maximal static frictional force on any wheel is $\mu_s mg/4$ where m is the total mass of the car. Thus the maximum acceleration of the car without any wheel slipping is $\mu_s g/4$ (more wheels would lower this). This means the maximum speed for driving around the curve is $v_{max} = \sqrt{\mu_s g r}/2 = \sqrt{0.2 * 9.82 * 15}/2 = 2.7$m/s (9.8 km/h to 2 sig. fig.).

b) Find the maximum torque from the engine that can be applied to the two front wheels of radius 0.25m if the wheels are not to slip when driving up a slope of 10%, again assuming that the weight of the car is equally distributed over all four wheels.

Solution: The torque from the engine must be offset by the static friction if the wheels are not to slip. The maximum static friction on each wheel is given by $f_{s,max} = \mu_s N = \mu_s (m/4) g \cos\theta$. The torque from the engine will generate a force at the edge of the wheel of $F_m = \tau/r$, so the maximum torque that can be applied to a single wheel is $\tau_{max} = \mu_s (m/4) g \cos\theta r$. Since there are two front wheels, the maximum torque the engine can supply without the front wheels slipping is $\tau_{max} = \mu_s (m/2) g \cos\theta r = 0.2 * (1200/2) * 9.82 * \cos(\arctan 0.1) * 0.25 = 290$Nm (2 sig.fig).

c) With no people in the car and no engine, the centre of mass of the car body is exactly in the middle of the car. Find the actual centre of mass of the car if there are two people of mass $m = 75$kg each, sitting in the front seats 0.45m forward from the centreline of the car (as seen from the side) and two people of mass $m = 75$kg each sitting in the back seat 0.55m back from the centreline and the engine in the front of mass $m = 160$kg, located a distance 1.5m from the centre. The total mass of the car including engine and people is still $m = 1200$kg.

To simplify this calculation, you may assume that the centre of mass of the empty car, the people and the engine all lie at the same height above the ground.

Solution: By assumption, the masses can all be taken to lie in a straight line. The centre of mass is just the position weighted sum of the masses $\overrightarrow{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M} \int \overrightarrow{\mathbf{r}}\, \mathrm{d}m$. Taking the centre of the car to be the origin $r = 0$, the position of the centre of mass is $r_{\mathrm{CM}} = \left(m_{engine} d_{engine} + m_{front\ people} d_{front\ people} - m_{back\ people} d_{back\ people}\right)/M_{total} = (160 * 1.5 + 2 * 75 * 0.45 - 2 * 75 * 0.55)/1200 = 0.19$m, forward of the centre of the car.

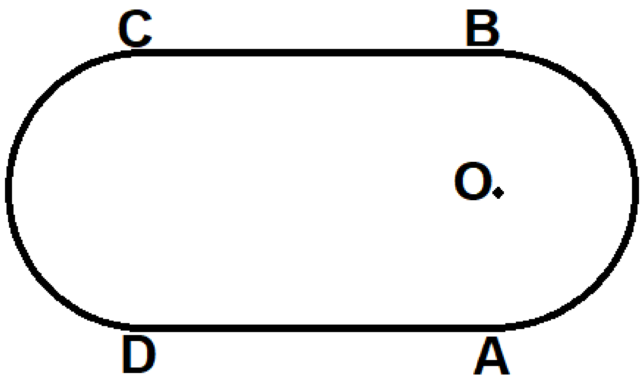
d) The front wheels are in line with the engine and the back wheels are 2.4m behind the front wheels. Draw a diagram of the vertical forces on the car when the car is on a horizontal road and find the normal forces on the front wheels and the back wheels (they may be different), assuming static equilibrium.

To simplify this calculation, you may assume that the wheels are attached to the car at the same height as the centre of mass of the car and that the normal force on the wheels is transfered directly to the car body at this height. You may neglect the mass of the wheels.

Solution: The total vertical forces must equal to zero because the car is not accelerating vertically. Thus $2N_{front} + 2N_{back} = mg$. In addition,

the total torque on the car must be zero around any point because the angular acceleration of the car is zero. The various masses of the car can be taken to all act at the centre of mass. The front wheels are a distance $(1.5 - 0.19)$m in front of the centre of mass and the back wheels are a distance $(2.4 - 1.5 + 0.19)$m behind the centre of mass (the back wheels are closer to the centre of mass). The total torque around the centre of mass of the car is then $(1.5 - 0.19) * 2 * N_{front} - (2.4 - 1.5 + 0.19) * 2 * N_{back} = 0$. This gives two equations for the two unknown normal forces. Solving this gives $N_{front} = 2700$N and $N_{back} = 3200$N, slightly different from the $mg/4 = 2900$N if the weight is equally distributed on all four wheels.

Problem 3: Athletics track



An athletics track consists of two straight sections (B to C and D to A in the figure) each of length $d = 100$m and two curved sections (A to B and C to D in the figure) formed as half circles with radius r that connect the two straight sections. The curved sections are also 100m in length, such that the total length of the track is 400m.

a) A runner of mass $m = 79$kg, starting from rest at the beginning of the curved section at point A, accelerates at a constant rate and takes $t = 12$s to cover the 100m of the curved section to point B. Find the angular acceleration of the runner relative to an axis passing vertically through the point at the centre of the half circle of the curved section (point O in the figure).

Solution: Use the angular kinematic relation $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$. The initial angular velocity is $\omega_i = 0$ and the runner runs through an angle $\theta_f - \theta_i = \pi$, so $\alpha = 2 * (\theta_f - \theta_i)/t^2 = 2\pi/12^2 = 0.044\text{s}^{-2}$ (2 sig. fig.).

b) Explain whether the angular momentum of the runner is conserved when they run around this curved section from A to B.

Solution The angular momentum of the runner is not conserved during this part. In the formula $\vec{L} = \vec{r} \times \vec{p}$ the magnitude of $\vec{p}$ is changing because the speed is increasing. Alternatively, the moment of inertia around point O is constant, just $I = mr^2$, but the angular velocity is changing because the angular acceleration is not zero.

c) The runner now runs along the straight section from B to C for 100m at a constant speed $v = 9.6\text{ms}^{-1}$. Find their angular momentum with respect to the same point as part (a), (point O in the figure). Is this angular momentum conserved?

Solution: The formula $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$ is most easily used in its lever arm form where the lever arm is just the distance r (nearest approach of the straight line through B and C to the point O) . Thus $\vec{\mathbf{L}} = rp = rmv = \left(\frac{100}{\pi}\right) * 79 * 9.6 = 24000$ kg m^2 s^{-1} (2 sig. fig.) Thus the angular momentum is constant (and non-zero) over the straight section from B to C.

d) If the runner now runs around the second curved section from C to D at the same constant speed v, show that relative to the same point in parts (a) and (c) (point O in the figure), when the runner has run a distance s along the curved section from point C, a net torque must be acting on the runner whose vertical component is

$$\tau = \frac{mdv^2}{r} \cos\left(\frac{s}{r}\right)$$

Solution: We want to find the torque $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$. Because the runner is not going in a straight line at a constant speed, they have an acceleration and this acceleration is directed towards the centre of the half circle they are running around, with magnitude $a = v^2/r$ (uniform circular motion). This acceleration (and net force) is not directed towards the point O, so the cross product in the torque formula is not zero.

An easy way to evaluate the torque formula is to write the vector between the runner and O as $\vec{\mathbf{r}} = \vec{\mathbf{d}} + \vec{\mathbf{R}}$ where $\vec{\mathbf{d}}$ is the vector from O to the centre of the half circle C to D and $\vec{\mathbf{R}}$ is the vector from the centre of this circle to the runner. Then the cross product becomes $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \left(\vec{\mathbf{d}} + \vec{\mathbf{R}}\right) \times m\vec{\mathbf{a}} = m\vec{\mathbf{d}} \times \vec{\mathbf{a}} = \frac{mdv^2}{r} \sin\theta$ where θ is the angle between $\vec{\mathbf{d}}$ and $\vec{\mathbf{a}}$. By trigonometry, when θ is measured in radians, $\sin\theta = \cos\left(\frac{s}{r}\right)$ because $\theta = \frac{s}{r} + \frac{\pi}{2}$.

Problem 4: Blocks on a spring

The equation for a damped harmonic oscillator is

$$y(t) = Ae^{-\frac{bt}{2m}} \sin(\omega t + \phi)$$

A light cube of mass $m = 0.24$kg and side length $l = 0.030$m is attached vertically below a much more massive block of mass $M = 420$kg by a spring of spring constant $k = 31$N/m and unstressed equilibrium length $x_0 = 0.11$m. The larger block is held at rest such that its lower edge is a height $h = 2.1$m above the ground.

a) Find the natural frequency of the oscillations of the cube-spring system. State any simplifying assumptions you make.

Solution: We can assume that the larger block does not move and treat the lighter block as a simple harmonic system. Substituting into Newton's 2nd law (or dimensional analysis) shows the natural frequency is given by $\omega = \sqrt{k/m} = \sqrt{31/0.24} = 11$s^{-1} (2 sig. fig.)

b) Find the total equilibrium length of the spring when the cube is attached below the heavier block (there is gravity).

Solution: The equilibrium length occurs when the weight of the lighter block downwards equals the Hooke's law spring force upwards $mg = kx$. So the total equilibrium length of the spring is $x_0 + x = x_0 + \frac{mg}{k} = 0.11 + \frac{0.24*9.82}{31} = 0.19$m (2 sig. fig.).

c) The cube is now displaced downwards so that its top edge is a distance $d = 0.034$m from this equilibrium position and then it is released from rest. Neglecting air resistance and other dissipative forces, find an equation for the position of the top of the cube above the ground, giving the numerical value of all parameters.

Solution: Neglecting air resistance and other dissipative forces means we can set $b = 0$ in the formula for a damped harmonic oscillator. If the block is released from rest, its displacement is equal to the amplitude A of the oscillations around the equilibrium position. Putting $t = 0$ at the start, we need $\sin\phi = 1$ and thus $\phi = \pi/2$. The height of the equilibrium position of the spring is $2.1 - 0.19 = 1.9$m above the ground. So the height of the top edge of the light block above ground is given by

$$y(t) = 1.9 - 0.034\cos(11t)$$

d) If instead of remaining at rest, the larger block is allowed to fall freely from the height of $h = 2.1$m with the cube attached by the spring vertically below it as in part (c), show that it will take a time t_h for the cube to hit the ground, where

$$h - x_0 - A\cos(\omega t_h) = l + \frac{mg}{k} + \frac{1}{2}gt_h^2$$

Solution: The equilibrium point of the spring starts out at a distance $h - x_0 - \frac{mg}{k}$ above the ground and the bottom edge of the smaller block is a distance l below this, so it starts out at a height $h - x_0 - \frac{mg}{k} - l$. After the larger block is released, the centre of mass will fall at an acceleration of g. The distance it falls from its initial position is given by $\frac{1}{2}gt^2$. The cube executes simple harmonic motion around the spring equilibrium position with a displacement $A\cos(\omega t)$. The height of the bottom edge of the smaller block can be approximated by $y(t) = h - x_0 - \frac{mg}{k} - l - A\cos(\omega t) - \frac{1}{2}gt^2$. Setting this height equal to zero (the ground) at time t_h and rearranging, gives the result. The equivalence principle (not part of FYS100) can be applied by absorbing the factor of $\frac{mg}{k}$ into the amplitude A since in free fall the equilibrium length of the spring is x_0.

FYS100 Physics – Formula sheet

Rotational motion about a fixed axis	**Translational motion**
Angular velocity $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Translational velocity $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Angular acceleration $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Translational acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Net torque $\sum_k \tau_k = I\,\alpha$	Net force $\sum_k F_k = m\,a$
$\alpha = \text{constant} \begin{cases} \omega_f = \omega_i + \alpha\,t \\ \theta_f = \theta_i + \omega_i\,t + \frac{1}{2}\alpha\,t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\,t \end{cases}$	$a = \text{constant} \begin{cases} v_f = v_i + a\,t \\ x_f = x_i + v_i\,t + \frac{1}{2}a\,t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\,t \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Work $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Rotational kinetic energy $K = \frac{1}{2}\,I\,\omega^2$	Kinetic energy $K = \frac{1}{2}\,m\,v^2$
Power $\mathcal{P} = \tau\,\omega$	Power $\mathcal{P} = F\,v$
Angular momentum $L = I\,\omega$	Linear momentum $p = m\,v$
Net torque $\sum_k \tau_k = \frac{\mathrm{d}L}{\mathrm{d}t}$	Net force $\sum_k F_k = \frac{\mathrm{d}p}{\mathrm{d}t}$

General formulas	
Motion with constant acceleration	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\,t + \frac{1}{2}\,\vec{\mathbf{a}}\,t^2 \end{cases}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = m\,\vec{\mathbf{a}}$
Work	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Work-kinetic energy theorem	$\Delta K = W$
Linear momentum	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newton's second law	$\sum_k \vec{\mathbf{F}}_k = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$
Impulse	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impulse-momentum theorem	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Center of mass	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\,\mathrm{d}m$
Moment of inertia	$I = \int r^2\,\mathrm{d}m$
Parallel-axis theorem	$I = I_{\text{CM}} + M\,D^2$
Torque	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Angular momentum	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Net torque	$\sum_k \vec{\boldsymbol{\tau}}_k = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$
Rotational motion	$s = r\,\theta,\ v = r\,\omega,\ a_c = r\,\omega^2,\ a_t = r\,\alpha$
Harmonic oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\ x(t) = A\cos(\omega t + \phi)$

Mathematical rules

Vector relations

Scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \, \cos\phi$
Magnitude of vector product	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert \, \lvert\vec{\mathbf{B}}\rvert \, \sin\phi$

Trigonometry

Definitions	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identities	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Derivatives	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$

Quadratic equations

Equation	$a\,t^2 + b\,t + c = 0$
Solution	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$

Equation of a straight line

Two points on the line are given	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$

Oppgave 1: Elektrisk fly

Et propellfly med masse $m_p = 2,4$ tonn flyr horisontalt i en høyde av $A = 2300\text{m}$.

a) Tegn et diagram som viser retningen til relevante krefter på flyet som holder det flyvende i konstant høyde med konstant hastighet.

b) Hvis forholdet mellom løftekraft og dragkraft for flyet er $R = 11$, finn skyvekraften og effekten som trengs fra motoren for å holde flyet horisontalt med en konstant hastighet på $v = 220\text{km/t}$.

c) Propellen drives av en elektrisk motor med et elektrisk batteri med massetetthet på $\rho_m = 2400$ kg/m^3 og en fulladet energitetthet på $\rho_e = 1800$ MJ/m^3. Ved å betrakte forskjellige størrelser av batteriet, vis at det er en maksimal avstand flyet kan drives med konstant høyde og hastighet, gitt av

$$d = \frac{\rho_e}{\rho_m}\frac{R}{g}$$

Oppgi eventuelle antakelser du gjør.

d) Estimer verdien av denne avstanden ved å bruke verdiene gitt ovenfor. Hvordan kan situasjonen være annerledes i den virkelige verden?

Oppgave 2: Isete vei

a) En forhjulsdrevet bil med masse $m = 1200\text{kg}$ kjører på en isete vei. Den statiske friksjonskoeffisienten mellom dekkene og den isete veien er $\mu_s = 0,20$ og koeffisienten for kinetisk friksjon er $\mu_k = 0,15$. Forutsatt at vekten av bilen er jevnt fordelt på alle fire hjulene, hva er den maksimale hastigheten bilen kan kjøre rundt en horisontalkurve med radius $r = 15\text{m}$ uten at noen av hjulene glir?

b) Finn det maksimale dreiemomentet fra motoren som kan påføres de to forhjulene med radius $0,25\text{m}$ hvis hjulene ikke skal skli når bilen kjører opp en bakke på 10%, igjen forutsatt at vekten av bilen er likt fordelt over alle fire hjul.

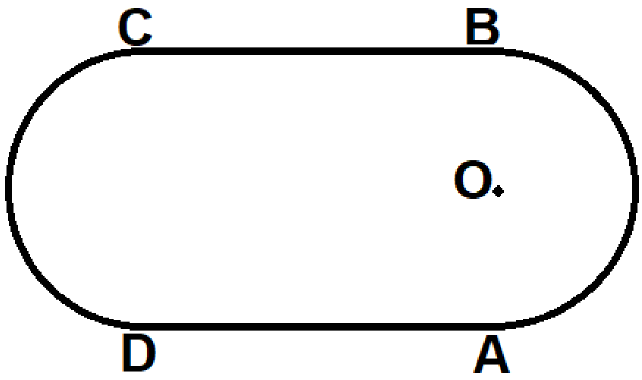
c) Uten personer i bilen og ingen motor, er massesenteret til bilen nøyaktig i midten av bilen. Finn det faktiske massesenteret til bilen hvis det er to personer med masse $m = 75\text{kg}$ hver, som sitter i forsetene $0,45\text{m}$ fremover fra bilens senterlinje (sett fra siden) og to personer med masse $m = 75\text{kg}$ hver, sittende på baksetet $0,55\text{m}$ bak fra senterlinjen og motoren foran med masse $m = 160\text{kg}$, plassert en avstand $1,5\text{m}$ fra bilens senter. Den totale massen til bilen inkludert motor og personer er fortsatt $m = 1200\text{kg}$.

For å forenkle denne beregningen kan du anta at massesenteret til den tomme bilen, personene og motoren ligger i samme høyde over bakken.

d) Forhjulene er på linje med motoren og bakhjulene er $2,4\text{m}$ bak forhjulene. Tegn et diagram over de vertikale kreftene på bilen når bilen er på horisontal vei og finn normalkreftene på forhjulene og bakhjulene (de kan være forskjellige), forutsatt statisk likevekt.

For å forenkle denne beregningen kan du anta at hjulene er festet til bilen i samme høyde som bilens massesenter og at normalkraften på hjulene overføres direkte til bilens karosseri i denne høyden. Du kan neglisjere massen på hjulene.

Oppgave 3: Friidrettsbane



En friidrettsbane består av to rette seksjoner (B til C og D til A på figuren) hver med lengde $d = 100$m og to buede seksjoner (A til B og C til D på figuren) dannet som halvsirkler med radius r som forbinder de to rette seksjonene. De buede seksjonene er også 100m lange, slik at den totale lengden på banen er 400m.

a) En løper med masse $m = 79$kg, starter fra hvile ved begynnelsen av den buede seksjonen ved punkt A, løper med konstant akselerasjon og bruker $t = 12$s for å dekke de 100m av det buet snittet til punkt B. Finn vinkelakselerasjonen til løperen i forhold til en akse som går vertikalt gjennom punktet i sentrum av det buede snittets halvsirkel (punkt O på figuren).

b) Forklar om drivmomentet til løperen er bevart når de løper rundt den buede delen fra A til B.

c) Løperen løper nå langs de 100m fra B til C med konstant fart $v = 9,6\text{ms}^{-1}$. Finn deres drivmoment i forhold til samme punkt som i del (a), (punkt O i figuren). Er dette drivmomentet bevart?

d) Hvis løperen nå løper rundt den andre buede seksjonen fra C til D med samme konstant fart v, vis at i forhold til samme punkt i delene (a) og (c) (punkt O i figuren), når løperen har løpt en strekning s langs den buede seksjonen fra punkt C, må et netto dreiemoment virke på løperen hvis vertikale komponent er

$$\tau = \frac{mdv^2}{r} \cos\left(\frac{s}{r}\right)$$

Oppgave 4: Blokker på en fjær
Ligningen for en dempet harmonisk oscillator er

$$y(t) = Ae^{-\frac{bt}{2m}} \sin(\omega t + \phi)$$

En kube med masse $m = 0,24$kg og sidelengde $l = 0,030$m er festet vertikalt under en mye mer massiv masseblokk $M = 420$kg med en fjær med fjærkonstant $k = 31$N/m og ubelastet likevektslengde $x_0 = 0,11$m. Den større blokken holdes i ro slik at dens nedre kant er en høyde $h = 2,1$m over bakken.

a) Finn egenfrekvensen til oscillasjonene til kube-fjærsystemet. Oppgi eventuelle forenklende forutsetninger du gjør.

b) Finn den totale likevektslengden til fjæren når kuben er festet under den tyngre blokken (det er gravitasjon).

c) Kuben er nå forskjøvet nedover slik at dens øverste kant er en avstand $d = 0,034$m fra denne likevektsposisjonen og deretter frigjøres fra hvile. Ved å neglisjere luftmotstand og andre dissipative krefter, finn en ligning for posisjonen til toppen av kuben over bakken, og gi den numeriske verdien for alle parametere.

d) I stedet for å forbli i ro, hvis den større blokken faller fritt fra høyden $h = 2,1$m med kuben festet med fjæren vertikalt under den som i del (c), vis at kuben vil treffe bakken etter en tid t_h, hvor

$$h - x_0 - A\cos(\omega t_h) = l + \frac{mg}{k} + \frac{1}{2}gt_h^2$$

FYS100 Fysikk – formelark

Rotasjon om en fast akse	**Éndimensjonal bevegelse**
Vinkelhastighet $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$	Hastighet $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
Vinkelakselerasjon $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$	Akselerasjon $a = \frac{\mathrm{d}v}{\mathrm{d}t}$
Resultantmoment $I\,\alpha = \sum_k \tau_k$	Resultantkraft $m\,a = \sum_k F_k$
$\alpha = \text{konstant} \begin{cases} \omega_f = \omega_i + \alpha\, t \\ \theta_f = \theta_i + \omega_i\, t + \frac{1}{2}\alpha\, t^2 \\ \omega_f^2 = \omega_i^2 + 2\,\alpha\,(\theta_f - \theta_i) \\ \theta_f = \theta_i + \frac{1}{2}\,(\omega_i + \omega_f)\, t \end{cases}$	$a = \text{konstant} \begin{cases} v_f = v_i + a\, t \\ x_f = x_i + v_i\, t + \frac{1}{2} a\, t^2 \\ v_f^2 = v_i^2 + 2\,a\,(x_f - x_i) \\ x_f = x_i + \frac{1}{2}\,(v_i + v_f)\, t \end{cases}$
Arbeid $W = \int_{\theta_i}^{\theta_f} \tau\,\mathrm{d}\theta$	Arbeid $W = \int_{x_i}^{x_f} F\,\mathrm{d}x$
Kinetisk energi $K = \frac{1}{2}\, I\,\omega^2$	Kinetisk energi $K = \frac{1}{2}\, m\, v^2$
Effekt $\mathcal{P} = \tau\,\omega$	Effekt $\mathcal{P} = F\, v$
Spinn $L = I\,\omega$	Bevegelsesmengde $p = m\, v$
Spinnsatsen $\frac{\mathrm{d}L}{\mathrm{d}t} = \sum_k \tau_k$	Newtons 2. lov $\frac{\mathrm{d}p}{\mathrm{d}t} = \sum_k F_k$

Generelle sammenhenger	
Bevegelse med konstant akselerasjon	$\begin{cases} \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\, t \\ \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\, t + \frac{1}{2}\, \vec{\mathbf{a}}\, t^2 \end{cases}$
Newtons 2. lov	$m\,\vec{\mathbf{a}} = \sum_k \vec{\mathbf{F}}_k$
Arbeid	$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}$
Arbeid-kinetisk energi teoremet	$\Delta K = W$
Bevegelsesmengde	$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$
Newtons 2. lov	$\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \sum_k \vec{\mathbf{F}}_k$
Impuls	$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,\mathrm{d}t$
Impuls-bevegelsesmengde teoremet	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
Massesenter	$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M} \int \vec{\mathbf{r}}\,\mathrm{d}m$
Treghetsmoment	$I = \int r^2\,\mathrm{d}m$
Steiners sats (parallellakseteoremet)	$I = I_{\text{CM}} + M\, D^2$
Kraftmoment	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Spinn	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Spinnsatsen	$\frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t} = \sum_k \vec{\boldsymbol{\tau}}_k$
Sirkelbevegelse	$s = r\,\theta,\; v = r\,\omega,\; a_c = r\,\omega^2,\; a_t = r\,\alpha$
Harmonisk oscillator	$\frac{d^2x}{dt^2} = -\omega^2 x,\; x(t) = A\cos(\omega t + \phi)$

Matematiske sammenhenger

Vektorrelasjoner	
Prikkprodukt	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert \cos\phi$
Absoluttverdi av kryssprodukt	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = \lvert\vec{\mathbf{A}}\rvert\,\lvert\vec{\mathbf{B}}\rvert \sin\phi$
Trigonometri	
Definisjoner	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
Identiteter	$\sin^2\alpha + \cos^2\alpha = 1$
	$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\,\cos^2\alpha - 1 = 1 - 2\,\sin^2\alpha$
	$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$
	$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$
	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$
	$a^2 + b^2 - c^2 = 2ab\cos\gamma$
Deriverte	$\frac{\mathrm{d}\sin\alpha}{\mathrm{d}\alpha} = \cos\alpha$
	$\frac{\mathrm{d}\cos\alpha}{\mathrm{d}\alpha} = -\sin\alpha$
2. grads ligning	
Ligning	$a\,t^2 + b\,t + c = 0$
Løsning	$t = \dfrac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a}$
Ligningen for en rett linje	
Gitt to punkter på linjen	$y = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$