LØSNINGSFORSLAG H21

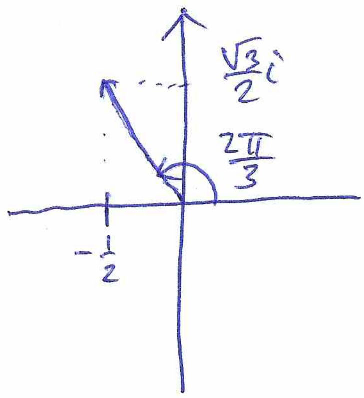
1 a) $z = 5+2i$, $w = -2+i$

$$z^2 = (5+2i)^2 = 5^2 + 2\cdot 5\cdot 2i + (2i)^2$$
$$= 25+20i-4 = \underline{\underline{21+20i}}$$

$$\frac{w}{z} = \frac{(-2+i)(5+2i)}{(5-2i)(5+2i)} = \frac{-10-4i+5i+2i^2}{5^2+2^2}$$
$$= \frac{-10+i-2}{29} = \underline{\underline{-\frac{12}{29} + \frac{i}{29}}}$$

$$|z|^2 = (5-2i)(5+2i) = 5^2+2^2 = \underline{\underline{29}}$$

b) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$



$$\left|-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right| = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = \underline{\underline{1}}$$

$$\Rightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2}i = 1\cdot e^{\frac{2\pi}{3}i}$$
$$= \underline{e^{\frac{2\pi}{3}i}}$$

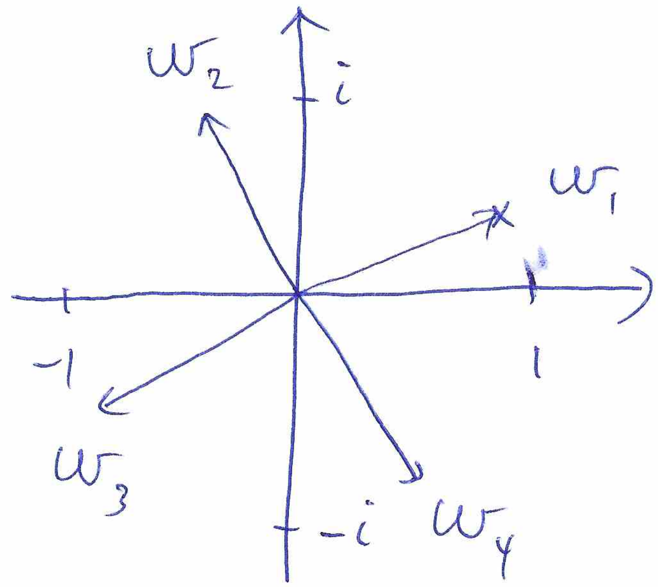
4. RØTTER:

$$w_1 = e^{\frac{2\pi}{3}\cdot\frac{1}{4}i} = e^{\frac{\pi}{6}i} = \underline{\underline{\frac{\sqrt{3}}{2} + \frac{1}{2}i}}$$

$$w_2 = e^{\frac{\pi}{6}i + \frac{2\pi}{4}i} = e^{\frac{4\pi}{6}i} = e^{\frac{2\pi}{3}i} = \underline{\underline{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}}$$

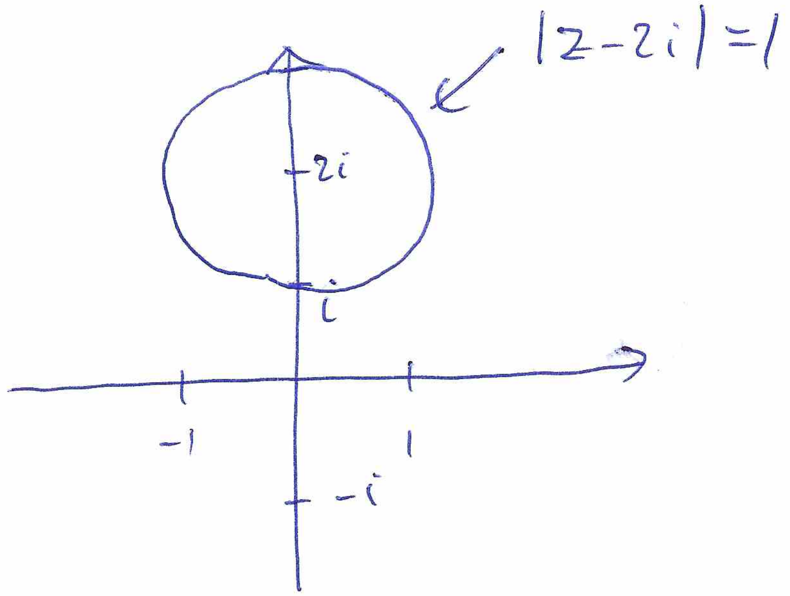
$$w_3 = e^{\frac{\pi}{6}i + \frac{4\pi}{4}i} = e^{\frac{7\pi}{6}i} = \underline{\underline{-\frac{\sqrt{3}}{2} - \frac{1}{2}i}}$$

$$w_4 = e^{\frac{\pi}{6}i + \frac{6\pi}{4}i} = e^{\frac{10\pi}{6}i} = e^{\frac{5\pi}{3}i} = \underline{\underline{\frac{1}{2} - \frac{\sqrt{3}}{2}i}}$$



c) $|z-2i|=1$

DETTE BETYR AT AVSTANDEN FRA Z TIL 2i SKAL VÆRE $=1$. ALTSÅ ER LØSNINGENE I EN SIRKEL MED SENTRUM I 2i, RADIUS=1



② a) $\int (3\sqrt{x} - 5e^{7x})dx = 3\cdot\frac{2}{3}x^{3/2} - \frac{5}{7}e^{7x} + C$

$$= \underline{\underline{2x^{3/2} - \frac{5}{7}e^{7x} + C}}$$

b) $\int \frac{\sin x}{\sqrt{2+3\cos x}}dx$

$u = 2+3\cos x$
$du = -3\sin x\,dx$
$-\frac{1}{3}du = \sin x\,dx$

$$= \int \frac{-\frac{1}{3}du}{\sqrt{u}} = -\frac{1}{3}\cdot 2\sqrt{u} + C$$

$$= \underline{\underline{-\frac{2}{3}\sqrt{2+3\cos x} + C}}$$

c) $\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} \, dx$

$$\boxed{\begin{aligned} u' &= x^2 \\ u &= \frac{1}{3}x^3 \\ v &= \ln x \\ v' &= \frac{1}{x} \end{aligned}}$$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

d) $\int \frac{2x^2+7}{(x+2)^2(x-1)} \, dx$

DEL BRØK OPPSPALTING:

$$\frac{2x^2+7}{(x+2)^2(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)}$$

$$= \frac{A(x+2)(x-1) + B(x-1) + C(x+2)^2}{(x+2)^2(x-1)}$$

MÅ MATCHE:

$$2x^2+7 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$$

$\underline{x=1}$: $2 \cdot 1^2 + 7 = A \cdot 0 + B \cdot 0 + C \cdot 3^2$

$9 = 9C \Rightarrow \underline{C=1}$

$\underline{x=-2}$: $2 \cdot (-2)^2 + 7 = A \cdot 0 + B \cdot (-3) + C \cdot 0$

$15 = -3B \Rightarrow \underline{B=-5}$

$\underline{x=0}$: $7 = A\cdot 2\cdot(-1) + B\cdot(-1) + C\cdot 4$

$7 = -2A + 5 + 4 \Rightarrow \underline{A=1}$

$$\int \frac{2x^2+7}{(x+2)^2(x-1)}\,dx = \int \left(\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} \right) dx$$

$$= \int \left(\frac{1}{x+2} - \frac{5}{(x+2)^2} + \frac{1}{x-1} \right) dx$$

$$= \underline{\underline{\ln|x+2| + \frac{5}{(x+2)} + \ln|x-1| + C}}$$

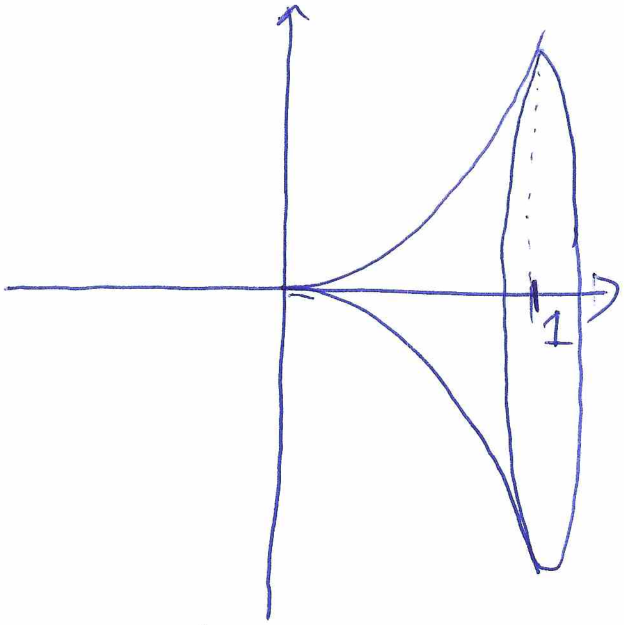
e) $\int 2x^3 \sqrt{1+36x^4}\,dx$

$$\boxed{\begin{aligned} u &= 1+36x^4 \\ du &= 4\cdot 36x^3\,dx \\ &= 144x^3\,dx \end{aligned}}$$

$$= \int \sqrt{u}\cdot \frac{2}{144}\,du = \frac{1}{72}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \underline{\underline{\frac{1}{108}\sqrt{(1+36x^4)^3} + C}} = \underline{\underline{\frac{1}{108}(1+36x^4)^{3/2} + C}}$$

③



$f(x) = 2x^3$

a) $$V = \pi \int_0^1 (f(x))^2 \, dx = \pi \int_0^1 (2x^3)^2 \, dx$$

$$= \pi \int_0^1 4x^6 \, dx = 4\pi \cdot \frac{1}{7} x^7 \Big|_0^1$$

$$= \frac{4\pi}{7}(1^7 - 0^7) = \underline{\underline{\frac{4\pi}{7}}} \simeq 1.80$$

b) $$A = 2\pi \int_0^1 |f(x)| \sqrt{1 + (f'(x))^2} \, dx$$

$$= 2\pi \int_0^1 2x^3 \sqrt{1 + (6x^2)^2} \, dx = 2\pi \int_0^1 2x^3 \sqrt{1 + 36x^4} \, dx$$

(FRA 2e)) $$= 2\pi \cdot \frac{1}{108} (1 + 36x^4)^{3/2} \Big|_0^1 = \frac{\pi}{54} \left[(1 + 36 \cdot 1^4)^{3/2} - 1^{3/2} \right]$$

$$A = \frac{\pi}{54}\left(37^{3/2} - 1\right) = \frac{\pi}{54}\left(37\sqrt{37} - 1\right) \simeq 13.04$$

④ a) $y'' + 2y' + 10y = 0$

KAR. LIGN.

$$r^2 + 2r + 10 = 0$$

$$r = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{-2 \pm \sqrt{-36}}{2}$$

$$= \frac{-2 \pm 6i}{2} = -1 \pm 3i$$

GENERELL LØSN:

$$y_h = Ae^{-x}\cos 3x + Be^{-x}\sin 3x$$

b) $y'' + 2y' + 10y = x^2$

HOM. LØSNING FRA a).

PARTIKULÆR LØSNING:

$$y_p = ax^2 + bx + c$$

$$y_p' = 2ax + b$$

$$y_p'' = 2a$$

4b) FORTS:

$$y_p'' + 2y_p' + 10y_p = 2a + 2\cdot(2ax+b) + 10(ax^2+bx+c)$$

$$= 10ax^2 + (4a+10b)x + 2a+2b+10c$$

$$= x^2$$

SÅ:

x^2: $10a = 1 \quad \Rightarrow a = \frac{1}{10}$

x: $4a + 10b = 0$

1: $2a + 2b + 10c = 0$

$$b = -\frac{4a}{10} = -\frac{2}{5\cdot 10} = -\frac{1}{25}$$

$$c = \frac{-2a-2b}{10} = -\frac{\frac{1}{5} - \frac{2}{25}}{10} = -\frac{\frac{3}{25}}{10}$$

$$= -\frac{3}{250}$$

$$y_p = \frac{1}{10}x^2 - \frac{1}{25}x - \frac{3}{250}$$

$\Rightarrow$ LØSNING:

$$y = Ae^{-x}\cos 3x + Be^{-x}\sin 3x + \frac{1}{10}x^2 - \frac{1}{25}x - \frac{3}{250}$$

(5) $y\ln x + x^3 + y^3 = 9$, $x > 0$.

a) $\frac{d}{dx}(y\ln x + x^3 + y^3) = \frac{d}{dx}(9)$

$$\frac{dy}{dx}\ln x + y\cdot\frac{1}{x} + 3x^2 + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(\ln x + 3y^2) + 3x^2 + \frac{y}{x} = 0$$

$$\underline{\underline{\frac{dy}{dx} = -\frac{3x^2 + \frac{y}{x}}{\ln x + 3y^2}}} = -\frac{3x^3 + y}{x\ln x + 3xy^2}$$

b) VISE AT P(1,2) LIGGER PÅ KURVA:

VS: $2\cdot\ln 1 + 1^3 + 2^3 = 1 + 8 = \underline{9} = HS$

QED.

$$\left.\frac{dy}{dx}\right|_{(1,2)} = -\frac{3\cdot 1^2 + \frac{2}{1}}{\ln 1 + 3\cdot 2^2} = -\frac{3+2}{12} = -\frac{5}{12}$$

1. PUNKTS FORMEL:

$$y - y_1 = a(x - x_1)$$

$$y - 2 = -\frac{5}{12}(x - 1)$$

$$y = -\frac{5}{12} + \frac{5}{12} + 2$$

$$\Rightarrow y = -\frac{5}{12} + \frac{29}{12}$$

6

$r = 0.5$

$h_0 = 1$

$$\frac{dV}{dt} = -k\sqrt{h}$$

a) $V = \pi r^2 \cdot h$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} = -k\sqrt{h}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{k}{\pi r^2}\sqrt{h} \quad , \quad h(0) = h_0 = 1$$

SEPARABEC:

$$\frac{dh}{\sqrt{h}} = -\frac{k}{\pi r^2} dt$$

$$\int \frac{dh}{\sqrt{h}} = -\int \frac{k}{\pi r^2} dt$$

$$2\sqrt{h} = -\frac{k}{\pi r^2} t + C.$$

INITIAL BETINGELSE: $h(0) = 1$

$$\Rightarrow 2 \cdot \sqrt{h(0)} = -\frac{k}{\pi r^2} \cdot 0 + C \quad \Rightarrow \quad C = 2$$

$$2\sqrt{h} = -\frac{k}{\pi r^2} t + 2$$

$$\sqrt{h} = -\frac{k}{2\pi r^2} t + 1$$

$$\underline{\underline{h = \left(1 - \frac{k}{2\pi r^2} t\right)^2 =}}$$

b) $t = 10$ (1 MIN): $h(10) = \frac{1}{2} h(0) = \frac{1}{2}$

$$\Rightarrow \frac{1}{2} = \left(1 - \frac{k}{2\pi r^2} \cancel{t}0\right)^2$$

$$\frac{1}{\sqrt{2}} = 1 - \frac{k}{2\pi r^2} \cdot 10$$

$$\frac{10k}{2\pi r^2} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{k}{2\pi r^2} = \frac{1}{10}\left(1-\frac{1}{\sqrt{2}}\right) \simeq 0{,}0293$$

$$\Rightarrow k = 2\pi r^2 \cdot \frac{1}{10}\left(1-\frac{1}{\sqrt{2}}\right) \simeq 0{,}0460$$

TANKEN ER TOM NÅR

$$h(t) = 0 = \left(1-\frac{k}{2\pi r^2}t\right)^2$$

$$\Rightarrow 1-\frac{k}{2\pi r^2}t = 0$$

$$t = \frac{2\pi r^2}{k} = \frac{10}{1-\frac{1}{\sqrt{2}}} = \frac{10\sqrt{2}}{\sqrt{2}-1}$$

$$t \simeq 34{,}1 \text{ (MINUTTER)}$$