MAT300 VEKTORANALYSE – NOTES ON EXAM

AUTUMN 2021, UNIVERSITY OF STAVANGER

This document contains information about the **steps** required to solve each task in the exam. Note that all of the tasks (the ones labelled by the alphabet letters "a", "b", "c" or "d", not the sub-tasks that have the additional label "i", "ii" or "iii"...) are worth the same amount of points. In all exercises, for each required step that was done correctly (or semi-correctly in some cases), the student will get points.

Exercise 1(a).

1(a)i). Required steps:

- Compute $\frac{d\mathbf{r}_1(t)}{dt}$.
- Evaluate $\frac{d\mathbf{r}_1(t)}{dt}$ at $t = \frac{\pi}{2}$.

1(a)ii). Required steps:

- Compute $\left|\frac{d\mathbf{r}_1(t)}{dt}\right|$.
- Write integrand as a function of the parameter t (i.e. $\rho(\mathbf{r}_1(t))$).
- Write the correct expression of the line integral of a scalar field $\int_{\mathscr{C}} \rho(\mathbf{r}_1(t)) \left|\frac{d\mathbf{r}_1(t)}{dt}\right| dt$ (i.e. the mass of the piece of wire).
- Evaluate the integral.

Exercise 1(b).

1(b)i). Required steps:

- Write x and z in terms of $y = t$.
- Find the range of t (from the given end points of the curve).

1(b)ii). Required step:

- Carrying the method of finding the scalar potential ϕ for the vector field $\mathbf{F}$ correctly.

Remark:

- Even if your answer is wrong, you would get some points as long as you show that you have some grasp over the method.

1(b)iii). Required step:

- Compute line integral of $\mathbf{F}$ over $\mathscr{C}_2$ by evaluating ϕ on the **given** end points.

Remark:

- It is considered a valid answer to compute this line integral by using a parametrisation of $\mathscr{C}_2$ and **carrying all the steps correctly**.

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Exercise 2(a).

2(a)i). Required steps:

- Sketch the given curves.
- Find their points of intersection (the vertices).
- Correctly identify the region D in the xy-plane.

2(a)ii). Required steps:

- Write the correct range of x when the domain is treated as x-simple.
- Write the correct range of y when the domain is treated as x-simple.

2(a)iii). Required steps:

- Treat the integrand as a function of x **first** and correctly integrate it using the limits for x.
- Complete the computation by integrating the obtained expression using the limits of y.

Exercise 2(b).

2(b)i). Required step:

- Find the equation of the plane using the given data.

2(b)ii). Required steps:

- Say that the area is given by $\iint_{\mathscr{S}} dS$.
- Write the correct formula for dS.
- Compute dS.
- Evaluate the integral and find the area.

Exercise 3(a).

3(a)i). Required steps:

- Mention that the function is odd under the transformation $(x, y, z) \mapsto (-x, y, z)$ (i.e. odd with respect to the x-variable).
- Mention that the transformation $(x, y, z) \mapsto (-x, y, z)$ is a symmetry of T (or that T is symmetric with respect to the yz-plane).
- Conclude that the value of the integral over the $x > 0$ half of T cancels out with the integral over the $x < 0$ half and explain why.

3(a)ii). Required step:

- Compute the angle ϕ using any method.

Exercise 3(b). Required steps:

- Find the correct limits in the chosen coordinate system.
- Write the correct dV according to the chosen coordinate system.
- Correctly integrate the first term of the integrand, $3z$.
- Correctly integrate the second term of the integrand, $-4xy$ (it is obviously valid to just mention that $-4xy$ is odd with respect to x so its integral vanishes due to symmetry and then conclude).

Exercise 3(c)). Required steps:

- Write the correct formula of NdS.
- Compute NdS.
- Choose the correct sign of NdS and justify your choice.
- Give the flux integral as a double integral of x and y.
- specify the correct domain of integration (which is the projection of the surface) and give the correct integration limits in the chosen coordinate system.
- Evaluate the double integral.

Exercise 3(d). Required steps:

- Compute the gradient of f.
- Evaluate the gradient of f on the point $(-1, -1)$.
- Give a unit vector in the direction of the gradient.

Exercise 4(a). Required steps:

- Compute the divergence of F (scalar quantity).
- Compute the and curl of F (vector quantity).

Remarks:

- Writing the value of the divergence as a vector quantity wastes a few points.

Exercise 4(b).

4(b)i). Required steps:

- Describe the induced orientation on the boundary of the **open** surface.
- Give a valid explanation of the used method (see the solutions).

4(b)ii). Required steps:

- Give the correct direction of N_2.
- Give a valid explanation of the used method (see the solutions).

4(b)iii). Required steps:

- Use Stokes' theorem to write the line integral of F as a flux integral of $\nabla \times F$ over D.
- Give the correct unit normal vector field N_2.
- Write the flux integral as a double integral in x and y.
- Evaluate the integral.

Remark:

- It is also a valid answer to, instead of using Stokes' theorem, compute the line integral by using a correct parametrisation of $\mathscr{C}$ and carrying all the computation steps correctly (it takes a lot of time though).